Credit Ratings, Credit Crunches, and the Pricing of Collateralized Debt Obligations

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Abstract

We present a new model to shed light on why senior tranche spreads are relatively more exposed to macroeconomic growth shocks, while junior spreads are more exposed to credit availability shocks. We model a credit rating agency (CRA) that produces noisy ratings to maximize the proportion of firms with high ratings (Bayesian persuasion) but still ensures that firms choose lower risk projects. Increased fundamental volatility in bad times makes high-risk choices more appealing to firms, which the CRA responds to by increasing the precision of ratings. Only firms that can call existing bonds and issue new ones will choose low risk projects at such times. Therefore, the resulting high risk strategy for constrained firms in such periods implies that junior tranches get seriously impacted. In contrast, senior tranches are more exposed to growth shocks, which increase the risk of all firms’ projects. We structurally estimate the parameters of our macro-finance model and show that an endogenously generated “convexity effect”, in large part due to the time varying precision of credit ratings, is much more important in understanding CDO tranche spreads than the spread on the entire pool of firms, the subject of past studies.
1 Introduction

A typical structured debt product such as a collateralized debt obligation (CDO) is a large pool of economic assets with a prioritized structure of claims (tranches) against this collateral. These instruments have made it possible to repackage credit risks and produce claims with significantly lower default probabilities and higher credit ratings than the average asset in the underlying pool. The structured finance market demonstrated spectacular growth during the decade before the financial crisis of 2007/08 but almost dried up following massive downgrades and defaults of highly rated structured products during the crisis (see Coval, Jurek, and Stafford (2009b)). In an influential paper, Coval, Jurek, and Stafford (2009a) argue that investors did not adequately price the risk in senior CDO tranches prior to the financial crisis (see also Collin-Dufresne, Goldstein, and Yang (2012) and Wojtowicz (2014)). In this paper, we do not focus on mispricing at particular points of time, but provide a new theoretical model, which is based on the dynamic information content of credit ratings through macroeconomic and credit cycles. We then structurally estimate the parameters of this model, and show that it is able to explain a substantial proportion of the historical variation in CDO tranche spreads.

Following the work by these above authors, we study the time series of spreads on tranches on the Dow Jones North American Investment Grade Index of credit default swaps, which are shown in Figure 1. The “equity” tranche (top-left panel) represents the 0 to 3 percent loss attachment points (these securities suffer losses if the loss on the entire collateral pool is between 0 and 3 percent of the underlying capital, are wiped out if the losses exceed 3 percent), while the “senior” tranche (top-right panel) represents the 15 to 100 loss attachment points. While both spread series rose rapidly during the financial crisis, the rise in the senior tranche spread was more spectacular, from only about 10 basis points (b.p.) before the crisis, to above 230 b.p. at its peak. The equity tranche by comparison, only roughly doubled from its pre-crisis level of 1175 b.p to 2700 b.p. at its highest point. Post-crisis (2012-2014), the senior tranche spread was still 27 b.p., while the equity tranche spread returned to its pre-crisis level.

The bottom-left panel of Figure 1 shows the quarterly growth in real GDP between 2004 and 2014. As seen, GDP growth bottomed out in the middle of the great recession, and
resumed at a more normal pace soon after the recession. The bottom-right panel shows that the ratio of credit growth at nonfinancial companies to nominal GDP fell through the recession, and only bottomed out after 2-3 quarters of the end of the recession. Matching up with the tranche spreads in the top panels, the figures suggest that the senior tranche was more affected by the fluctuations in growth, while the equity tranche was more affected by credit growth fluctuations. We examine if this is true with some simple regressions.

In Table 1, we regress the spread on the entire pool (CDX) as well as different tranche spreads on the two fundamentals. For each of the spread series, it is noteworthy that despite the presence of a macroeconomic factor, credit growth additionally impacts tranche spreads. However, the relative importance of the two fundamentals for junior and senior spreads is quite different. In lines 4 to 6, we see that GDP growth only explains only about 14.5 percent of the variation in the equity tranche spread, while credit growth explains nearly 51 percent of its variation. Both variables are significant in a joint regression. In contrast, in lines 13 to 15, we see that GDP growth explains 56 percent of the variation in the senior tranche spread, but credit growth explains only about 18 percent. In this paper, we ask why the relative exposure of the junior and senior tranches to the alternative shocks is so different, and provide a new model to explain it.

There are three crucial ingredients in our model. First, we endogenize the risk of the firms using an asset substitution mechanism. In particular, firms optimally choose their risk based on the amount of debt that they need to service. Second, we introduce imperfect credit ratings using the Bayesian persuasion concept, which we discuss more completely below. This concept implies that the rating agency changes the intensity of its investigation of firms’ credit quality with the goal of maximizing the proportion of firms with high credit ratings. Finally, credit availability in the model can be in “available” or “nonavailable” states.

These features generate a mechanism that amplifies and propagates macroeconomic shocks and can create catastrophic risk observed in the prices of CDO tranches (see Collin-Dufresne, Goldstein, and Yang (2012)). According to this mechanism the rating agency produces a noisy signal (ratings) that allows the firms to borrow at the cost compatible with low-risk behaviour, i.e. the credit ratings abate the moral hazard problem just enough to induce low-risk behavior in current economic conditions. In a sense, this puts the firms on the
edge of low-risk and high-risk technologies and if economic conditions change the firms could switch to risky behavior. To prevent this switching the rating agency steps in and produces more precise signal (ratings). The new ratings can decrease the cost of borrowing for the firms to maintain low-risk behavior, if they can call existing debt. However, if credit availability is off, then, firms cannot call existing debt and will continue to choose high risk projects.

We incorporate these features into a model of CDO tranches, where firms’ bonds are pooled each period, and provide returns over 5 years, broken up in a short initial period of 1 year, after which the bonds can be called, and a longer period of 4 years, at the end of which the returns are distributed. In our model, we study how the information in credit ratings evolve over the business cycle and the pricing consequences of these dynamics. We apply our model to explain risk and pricing dynamics of the different CDO tranches. In particular we shed light on the difference in relative exposure of junior and senior tranches to macro and credit availability shocks.

Our paper builds on the coordinating role of rating agencies in driving better investment decisions by firms as in Boot, Milbourn, and Schmeits (2006) and Manso (2013). In both papers the models exhibit multiplicity of equilibria and the credit rating agency plays a coordinating role. In their work, ratings lower the cost of finance specially since certain classes of investors are forced by institutional rigidities to invest in highly rated securities. We instead build on the concept of Bayesian persuasion (exemplified in a litigation context in Kamenica and Gentzkow (2011)), in which the precisions of the ratings are controlled by the rating agencies investigation process. In good times, the agencies allow some degree of contamination of the good ratings class by conducting a less thorough examination of firms credit quality, but still ensuring that the overall cost of capital of the mix of firms is low enough to induce the low risk project choice by high quality firms. In periods of deteriorating fundamentals, the quality of the ratings are improved to weed out bad firms from the high rating class, so that once again good quality firms still pursue low risk projects. Overall, the procedure maximizes the amount of debt with high ratings. It is important to note that the time varying quality of ratings is distinct from alternative rating agency behaviors such as misreporting and ratings

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1We therefore have a coordination game with strategic complementarity as for example in Milgrom and Roberts (1990) and Cooper (1999).
inflation (see e.g. Fulghieri, Strobl, and Xia (2014)), which might also have played a role in financial crisis.

A significant contribution of our paper is a structural estimation of our Bayesian persuasion model. Our estimation proceeds in two stages. At the first stage we use standard maximum likelihood of regime switching models (see Hamilton (1994)) to estimate the cycles in credit availability and macroeconomic growth. The regimes are observed by the agents in the model, but are unobserved by the econometrician. In the second stage, we use the simulation method of moments (SMM) to estimate the parameters of firms’ projects that fit tranche spreads. Our estimated model provides several insights. In the model, during credit non-availability states, several firms cannot refinance their existing debt, and hence, they choose HR projects. Therefore an increase in risk of some firms (relative to credit availability states) implies that the chance of the equity tranches experiencing significant losses increases. But, since all firms do not increase their risk, the chance of all of them defaulting, an event that triggers losses in the senior tranche, does not increase. Instead, the spreads for senior tranches, are higher in low growth (R) states, where all firms’ volatility increases. This differential impact on senior and junior tranches helps our model match the different dynamics of these tranches, and in particular why junior tranches are relatively more exposed to credit shocks, and senior tranches to growth shocks. It is worth mentioning that as part of our specification of our model, investors require risk adjustments to the transition probabilities across different growth and credit availability states, which raise Q-measure or risk-adjusted default losses (credit spreads) even though we constrain project parameters for firms to match the historical low levels of default probability under the objective measure.

One of the key aspects of our model is the endogenously generated convexity effect of credit spreads. As was pointed out by David (2008), in structural form models of credit risk (such as the one presented here), credit spreads are convex function of firms’ asset values (capital stocks). Due to heterogeneity in firms’ capital accumulation, spreads for firms with low realized capital rise more dramatically, then for the fall of spreads of firms that have high

\[^2\]Bhamra, Kuehn, and Strebulaev (2009), Kuehn and Schmid (2014), Feldhutter and Schaefer (2015), Chen, Cui, He, and Milbrandt (2015), Christoffersen, Du, and Elkamhi (2013), and Culp, Nozawa, and Veronesi (2015) have also used this convexity effect to understand empirical properties of credit spreads.
realized capital. The greater the dispersion in capital stocks across firms, the greater is the difference in average spreads across firms, and the spread calculated for a representative firm with an average capital stock. In the model, heterogeneity increases in low growth states, but also to some extent when credit is unavailable. Therefore spreads increase in such states. The convexity effect not only implies an increase in the average spread generated by the model, but also the dynamics of spreads, as spreads increases in states with higher dispersion, which endogenously varies as the economy transitions through the macro and credit states. The convexity effect arises endogenously in our model as the credit rating agency changes the precision of its rating over time. By doing so, it affects the dispersion in borrowing costs across firms, which in turn affects their project choices, and the dispersion in their capital stocks. This is a feature not present in prior work on the convexity effect, such as in David (2008).

The remainder of the paper follows the following plan: Section 2 introduces the model. Section 3 analyzes the equilibria in the model with two different credit rating standards. Section 4 provides results on the pricing of CDO tranches. Section 5 presents empirical results. Section 6 concludes.

2 Model

The economy has a continuum of firms and investors and a monopolistic credit rating agency (CRA). The economy goes through macroeconomic cycles with two states, booms (B) and recessions (R), and credit cycles where either credit is available (state A) or not available (state N). The macro states are identified by GDP growth, which in a given period is distributed $N(\mu_i^g, \sigma_g)$, for $i \in \{B, R\}$. Credit availability states are identified by the ratio of credit growth of nonfinancial firms to GDP, which in a given period is distributed $N(\mu_i^c, \sigma_c)$, for $i \in \{A, N\}$ Overall, the composite states are $S \equiv \{BA, BN, RA, RN\}$, and are driven by a stationary Markov process with a $4 \times 4$ transition matrix under the objective measure $\Lambda \equiv (\lambda_{s,s'})$. Under the risk-neutral measure, the Markov transition matrix is $\Lambda^Q$, with elements
\( \lambda_{ss'} = \lambda_{s's'} \cdot e^{\beta_1 (\mu_s - \mu_{s'}) + \beta_2 (\sigma_s^2 - \sigma_{s'}^2)}, \) where \( \beta_1 \) and \( \beta_2 \), are the risk adjustment factor loadings for macroeconomic and credit state transitions, respectively.\(^3\)

**Firms:** There are two types of firms: *good* and *bad*. In each period a new pool of \( N \) bonds is created, with a constant proportion \( \hat{\alpha} \) of good firms. Each firm in the pool provides returns over three periods. In each period, every good firm chooses between two one-period projects: low risk, \( LR \), and high risk, \( HR \). A bad firm can only implement the \( HR \) project. There are no switching costs and a good firm could choose different projects in the first and second periods. Each project returns \( \tilde{r} \), which has a lognormal distribution with parameters \( \mu_p^s \) and \( \sigma_p^s \) for \( p \in \{LR, HR\} \) and \( s \in S \). The parameters depend only on the macro state, i.e. \( \mu_{p}^{BA} = \mu_{p}^{BN} \), \( \mu_{p}^{RA} = \mu_{p}^{RN} \), \( \sigma_{p}^{BA} = \sigma_{p}^{BN} \), \( \sigma_{p}^{RA} = \sigma_{p}^{RN} \) for \( p \in \{LR, HR\} \). Conditional on the state of the economy, the project returns across firms are independent, that is conditional firms’ risk is idiosyncratic. Each firm has capital in place \( k_t \), which evolves as \( k_{t+1} = \tilde{r}_{t+1} k_t \).

We assume that firms pay no dividends, and that each unit of capital is freely convertible into a unit of the numeraire good, i.e. the price of capital is one. Further, we assume that the choice of the project is not contractable, even though returns of the projects are observable ex-post. Since the returns have full support, the investors and the CRA cannot ex-post infer the true type of the project even though they update their beliefs about the type of the firm as described below.

We assume that at \( t = 0 \), each firm has capital in place \( \hat{K} \) and raises debt \( \hat{D} \) by issuing a two-period zero-coupon callable bond with call price \( H \). Therefore, total capital at \( t = 0 \) is \( K_0 = \hat{K} + \hat{D} \). We assume that the call price \( H = \hat{D} \), i.e. the bond can be called at par value. If at \( t = 1 \) credit is available, each firm can refinance its debt. In this case, the firm redeems the existing two-period bond and issues a new one-period bond in order to finance the call price of its existing bond. In case of default, the debt holders incur a proportional dead weight cost, \( \delta \), of the existing capital.

\(^3\)Such risk adjustments are required for models to simultaneously match the low average historical default rates of investment grade firms and the high level of their credit spreads in the “credit spreads puzzle” literature (see David (2008), Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Streubale (2009), and Chen (2010)).
**CRA:** Firms’ type is not observable by either investors or the CRA. The CRA however can conduct an investigation procedure whose results it reports truthfully to investors, and hence it can influence the beliefs of investors. Even though the investigation process is costless, the CRA can control its precision. In particular the $G$-rating could be assigned to a bad firm or the $B$-rating could be assigned to a good firm (although as we show below, the latter is never optimal). The type I and II errors associated with the ratings are $\nu \equiv \mathbb{P}[B|\text{good}]$ and $\pi \equiv \mathbb{P}[G|\text{bad}]$, respectively. As in Lizzeri (1999) and Kartasheva and Yilmaz (2012), we assume that the CRA commits to this structure of ratings. Investors’ beliefs about the type of a firm affects its cost of capital, and ultimately its project choice. For example, in periods when investors’ assess that the firm is less likely to be good, they charge a higher cost of capital, which leads even a good firm to choose the $HR$ project. In this case, the CRA can influence the investors’ beliefs by changing the precision of ratings, and based on the new ratings standards that it announces, investors update their beliefs about firms’ quality. Under the new beliefs, $G$-rated firms may refinance their debt at lower cost, and subsequently chose the $LR$ project.

We assume that the CRA attempts to issue as many $G$-ratings as possible. This preference for high ratings can result from institutional investment constraints, as is assumed in Boot, Milbourn, and Schmeits (2006). In particular, given a prior probability $\alpha$ that the firm is a good type, the CRA chooses $\nu$ and $\pi$ to maximize the unconditional probability of assigning the good rating $\mathbb{P}[G] = \alpha(1-\nu)+(1-\alpha)\pi$. The process of changing the precision of the investigation process to induce a particular outcome has been called “Bayesian persuasion” by Kamenica and Gentzkow (2011). The posterior investors’ beliefs are represented by the probabilities that the firm is good conditional on observing a $G$ or a $B$ rating, $\mathbb{P}[\text{good}|G]$ and $\mathbb{P}[\text{good}|B]$ respectively. There are two extreme cases. First, the CRA can perfectly separate good and bad firms choosing $\nu = 0$ and $\pi = 0$. Second, the CRA can produce completely uninformative ratings assigning $G$-rating to all firms, i.e. choosing $\nu = 0$ and $\pi = 1$. If what

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4 More generally, this assumption is consistent with the widespread view that the issuer-pays business model adopted by credit rating agencies leads to rating inflation (see e.g. Bar-Isaac and Shapiro (2011), Bolton, Freixas, and Shapiro (2012), Fulghieri et al. (2012), Kartasheva and Yilmaz (2012), Harris, Opp, and Opp (2013), and Cohn, Rajan, and Strobl (2013)).
follows, we denote $\alpha_0$ and $\alpha_1$ posterior beliefs that a firm is good if it gets the $G$-rating at $t = 0$ and if it retains the $G$-rating at $t = 1$ respectively.

**Investors:** Let $\hat{\alpha}_0$ be investors’ prior belief that any particular firm is good at $t = 0$. After observing the ratings, investors update their beliefs using Bayes’ law. In particular, their posterior belief satisfies

$$\alpha_0 \equiv P[\text{good} | G] = \frac{\hat{\alpha}_0 (1 - \nu)}{\hat{\alpha}_0 (1 - \nu) + (1 - \hat{\alpha}_0) \pi}.$$  

(1)

$$\chi_0 \equiv P[\text{good} | B] = \frac{\hat{\alpha}_0 \nu}{\hat{\alpha}_0 \nu + (1 - \hat{\alpha}_0)(1 - \pi)}.$$  

(2)

As we will see, given the face value of debt, $F$, investors are able to tell if G-rated firms optimally choose LR projects. Since the return distributions of the $LR$ and $HR$ projects are different, they are able to partially learn about the type of the firm by observing the realized returns of its project. Their updated belief satisfies

$$\hat{\alpha}_1(r) = \frac{\alpha_0 \phi_{LR}^s(r)}{\alpha_0 \phi_{LR}^s(r_1) + (1 - \alpha_0) \phi_{HR}^s(r_1)},$$  

(3)

where $\phi_p^s(r)$ is the probability density function of the return on $p$-type project in state $s$. If at $t = 0$, good firms choose the $HR$ project, the outcome of the project contains no information about the type of the firm and, therefore, $\hat{\alpha}_1 = \alpha_0$.

We assume that credit markets are perfectly competitive. Thus, in equilibrium the investors require return that yields them zero expected profit.

**Sequence of Events** At $t = 0$ the investors have prior beliefs $\hat{\alpha}_0$. The CRA chooses rating standard parameters and issues $G$- and $B$-ratings for all firms. After observing the ratings, the investors update their beliefs. If $\nu \geq \pi$, the investors’ beliefs that a firm is good increases for $G$-rated firms and decreases for $B$-rated firms. In what follows, we focus on the firms that obtain the $G$-rating. The investors’ beliefs for these firms increase from prior $\hat{\alpha}_0$ to $\alpha_0$. After obtaining a rating, each firm issues a two-period bond and starts a project. If a good firm chooses the $LR$ project at $t = 0$, the investors update their beliefs based on ((3)).

At $t = 1$, the CRA may adjust its ratings precisions, and investors update their beliefs according to (1) and (2). Since the bond is callable, if at $t = 1$ credit is available, firms can
refinance their debt. The refinancing decision will be discussed in the next section. At \( t = 1 \) firms initiate new projects as at \( t = 0 \). At \( t = 2 \), firms repay their debt. Figure 2 summarizes the sequence of events.

3 Equilibrium

This section describes a rational expectations equilibrium that arises in the model.

**Definition 1** An equilibrium is a set of strategies of the CRA, firms, and investors such that:

1. Good firms choose optimally between LR and HR projects at \( t = 0 \) and \( t = 1 \) in each state and decide whether to refinance their debt at \( t = 1 \) in the states when credit is available.

2. Investors earn zero expected profits under rational expectations about the type of the firm at \( t = 0 \) and \( t = 1 \), firms’ projects and refinancing decisions, and the CRA’s rating precision.

3. The CRA follows the Bayesian persuasion strategy.

**Equity value of firms known to be good:** To determine the project choices of good firms, we need to find the value to equity holders from each alternative. We use dynamic programming to determine a good firm’s equity value. At \( t = 2 \), the good firm’s equity value is

\[
E_2(K_2) = \max(K_2 - F, 0)
\]

where \( K_2 \) the accumulated capital, and \( F \) is the face value of debt to be repaid. The following lemma establishes the equity value at \( t = 1 \) given the project choice.

**Lemma 1** Suppose a good firm with capital \( K_1 \) chooses project \( p \) at \( t = 1 \) in state \( s \). Then the value its equity at \( t = 1 \) is

\[
E_1^s(K_1, p) = \sum_{s' \in S} \lambda_{ss'} \left[ K_1 e^{\mu_p s'} (0.5 (\sigma_p s')^2) N(-d_p s') - F N(-d_p s' - \sigma_p s') \right],
\]

where \( d_p s' = (\ln(F/K_1) - \mu_p s' - (\sigma_p s')^2)/\sigma_p s' \) and \( N(x) \) is the standard normal CDF.
At $t = 1$ in state $s$, the good firm chooses between the low and high risk projects

$$E^s_t(K_1) = \max_{P \in \{LR, HR\}} E^s_t(K_1, P).$$

At $t = 0$, the good firm chooses the project to maximize the equity value, i.e.

$$E^s_0(K_0) = \max_{P \in \{LR, HR\}} \left\{ \sum_{s' \in \mathcal{S}} \lambda_{ss'}^Q \mathbb{E} \left[ E^s_{t}(\tilde{r}_1 K_0) | P \right] \right\}.$$

Bad firms including those with the G-rating always implement the HR project.

**Credit ratings** At $t = 0$, the CRA issues ratings to every firm. After observing the ratings, the investors update their beliefs about the type of each firm. At $t = 1$ the investors further update their beliefs based on the outcomes of the projects. If the state changes, the CRA may adjust the ratings and induce another update of investors’ beliefs. The following lemma shows the CRA’s optimal choice of the rating standard in each period.

**Lemma 2** Suppose that prior to observing ratings investors have beliefs $\hat{\alpha}_t$ that a firm is good. Then to induce target level of beliefs $\alpha_t$ at either $t = 0$ or $t = 1$, the CRA chooses the following parameters of rating standard:

$$\nu = \mathbb{P}[B|\text{good}] = 0$$

$$\pi = \mathbb{P}[G|\text{bad}] = \frac{\hat{\alpha}_t(1 - \alpha_t)}{\alpha_t(1 - \hat{\alpha}_t)}.$$  \hspace{1cm} (7)

Solution (7) and (8) results in posterior beliefs such that the investors are certain that a firm is bad if it has the $B$-rating. Similar to Proposition 4 in Kamenica and Gentzkow (2011), it is optimal for the CRA to assign all good firms G ratings but mix some bad firms into the G rating. Expression (8) shows that conditional probability $\pi$ is decreasing in $\alpha_t$, that is higher posterior beliefs require less noisy ratings. At the same time, $\pi$ is increasing in $\hat{\alpha}_t$ meaning that higher prior beliefs allow the CRA to choose a looser rating standard. Since the firms with the $B$-ratings are all bad, the CRA never changes their ratings at $t = 1$.

The CRA adopts the following logic at $t = 1$ in state $s$. If under prior beliefs (and possible refinancing of its 2-period bond, to be discussed) the $G$-rated firms with capital $K_1$
chooses the \( LR \) project, the CRA keeps the ratings unchanged. Otherwise, if credit is available at \( t = 1 \), the CRA reevaluates firms with the \( G \)-ratings such that under updated beliefs
\[
E^s_1(K_1, LR) \geq E^s_1(K_1, HR), \tag{9}
\]
i.e. they prefer the LR project. We assume that if \( G \)-rated firms choose the \( HR \) project under the highest level of beliefs, i.e. \( \mathbb{P}[\text{good}|G] = 1 \), or credit is unavailable, the CRA perfectly separates good and bad firms and, therefore, increases the level of beliefs to unity. In this case, the rating parameters are \( \nu^s_1 = 0 \) and \( \pi^s_1 = 1 \). The rating accuracy implicitly depends on the level of firm’s capital \( K_1 \) and, thus, on firm’s leverage, since firms with higher leverage are more likely to choose the HR project. Therefore, at \( t = 1 \) there is a continuum of ratings indexed by firms’ level of capital and letter \( G(K_1) \) or \( B(K_1) \).

Similarly, at \( t = 0 \) in state \( s \), the CRA chooses rating parameters \( \nu^s_0 \) and \( \pi^s_0 \) as in Lemma 2 to induce beliefs \( \alpha_0 \), which is the minimum level of beliefs such that
\[
E^s_0(K_0, LR) \geq E^s_0(K_0, HR). \tag{10}
\]

**Debt value:** Due to limited liability if at maturity the value of firm’s capital is less than the face value of the bond, the value of the debt is the value of capital less bankruptcy costs. Therefore, at \( t = 2 \) the value of a bond belonging to a firm with capital \( K_2 \) is
\[
D_2(K_2) = \begin{cases} 
F & \text{if } F \leq K_2 \\
(1 - \delta)K_2 & \text{if } F > K_2,
\end{cases} \tag{11}
\]
where \( F \) is the bond’s face value. The following lemma gives the value of a bond at \( t = 1 \) in state \( s \) if investors know which project is going to be chosen.

**Lemma 3** Suppose a good firm with capital \( K_1 \) chooses project \( p \) at \( t = 1 \). Then, its value at \( t = 1 \) is
\[
D^s_1(K_1, p) = \sum_{s' \in S} \lambda^Q_{ss'} \left[ (1 - \delta)K_1 e^{\mu^s_p + 0.5(\sigma^s_p)^2} N(d^s_p) + F N(-d^s_p - \sigma^s_p) \right], \tag{12}
\]
where \( d^s_p \) is defined in the statement of Lemma 1.

\[ ^5 \text{Since the debt of the firm is fixed, the capital is a measure of the leverage of the firm. The leverage dependence of credit ratings is consistent with the ratings procedures used by most CRAs.} \]
Let $P_s^1(K_1) \in \{LR, HR\}$ be a good firm’s optimal project choice at $t = 1$ in state $s$. Then investors value of a $G$-rated firm’s bond is

$$D_s^1(K_1, G, \alpha_s^1) = \alpha_s^1 D_1^1(K_1, P_s^1(K_1)) + (1 - \alpha_s^1) D_1^1(K_1, HR).$$ (13)

Since investors do not observe the firm’s type, they take expectations of the value of bond conditional on its type. The value of a bond $D_s^1(K_1, B)$ belonging to a $B$-rated firm is given by (13) when $\alpha_s^1 = 0$.

The refinancing decision: A firm with capital $K_1$ and rating $Q_1 \in \{G, B\}$, can refinance its debt at $t = 1$ in state $s$ if credit is available. It can issues a one-period bond with face value $F_{12}^s$ such that $D_s^1(K_1, Q_1, \alpha_s^1) = H$. If $F_{12}^s < F_{02}$, then the firm can lower its borrowing costs. It is worth mentioning that a firm may be unable to refinance its debt if $H$ is greater than its debt capacity in that state. Given the log-normality of returns, the probability of the debt being repaid declines to zero as the face value increases. In particular, since we model bankruptcy costs, the value of the firm’s bond has a maximum value (its debt capacity) as we increase its face value. For a $G$-rated firm, the face value is decreasing in $\alpha_s^1$, as investors believe it is more likely to be a good type.

The value of the two-period bond at $t = 0$ depends on firms’ optimal decision on refinancing at $t = 1$. Let $R_s^s(K_1, Q_1, \alpha_s^1)$ be the payment made by the firm with capital $K_1$ and rating $Q_1$ to bondholders.

$$R_s^s(K_1, Q_1, \alpha_s^1) = \begin{cases} 
H & \text{if } F_{12}^s < F_{02} \\
D_1^1(K_1, Q_1, \alpha_s^1) & \text{otherwise,} 
\end{cases}$$ (14)

Then if $P_s^0 \in \{LR, HR\}$ is the firm’s optimal project choice at $t = 0$ in state $s$, the value of the two-period bond at $t = 0$ in state $s$ is

$$D_s^0(K_0) = \sum_{s' \in S} \lambda_{s,s'}^Q \left( \alpha_0^s \mathbb{E} \left[ R_s^s(\tilde{r}_1 K_0, G, \alpha_s^1) \mid P_0^s, s' \right] + (1 - \alpha_0^s) \mathbb{E} \left[ \pi_1^s(\tilde{r}_1 K_0) \ R_s^s(\tilde{r}_1 K_0, G, \alpha_s^1) \mid HR, s' \right] + (1 - \alpha_0^s) \mathbb{E} \left[ (1 - \pi_1^s(\tilde{r}_1 K_0)) \ R_s^s(\tilde{r}_1 K_0, B, 0) \mid HR, s' \right] \right),$$ (15)

where $\pi_1^s(K_1)$ is the probability that a bad firm with capital $K_1$ retains the $G$-rating at $t = 1$ in state $s'$. The first expectation in (15) corresponds to the value of the two-period bond of a
good firm. A bad firm rated G at \( t = 0 \) gets either G or B rating at \( t = 1 \). The bond of a bad firm that retains the G-rating at \( t = 1 \) has the same value as the bond of a good firm. The bond of a bad firm that gets downgraded at \( t = 1 \) has the value when investors are certain that the firm is bad. The second and third expectations in (15) provide the values for these two mutually exclusive events.

3.1 Belief Updating From Learning and Bayesian Persuasion

At \( t = 0 \) all good firms start with the same level of capital and debt, and hence choose the same project. At \( t = 1 \), updated beliefs of firms rated G being good, depend on the project return as shown in (3). Figure 3 shows the level of beliefs, \( P(\text{good}|G) \), before and after observing ratings at \( t = 1 \). As can be seen, the posterior of the firm being good, increases in the level of capital upto a range, since the mean return of the LR project exceeds that of the HR project. For higher levels of capital, the belief falls, as the relative likelihood of very high returns (capital) increase for the HR project, which has a higher variance.

If \( K_1 \) is sufficiently low (leverage of the firm is high), the asset substitution problem urges the firm to choose the HR project. In particular, if \( K_1 < K_{-1} \), the firm chooses the HR project even if investors are certain that the firm is good, i.e. \( P[\text{good}—G]=1 \), and the firm is able to refinance its debt. In this case, we assume that the CRA chooses perfectly precise ratings and therefore posterior beliefs, \( \alpha^s_1 \equiv P[\text{good}|G] = 1 \). On the contrary, if the level of capital is sufficiently high, i.e. \( K_1 \geq K_1 \), the firm chooses the LR project under any level of \( \alpha^s_1 \). In this case, the CRA does not adjust the ratings and \( \hat{\alpha}^s_1 = \alpha^s_1 \). Finally, if \( K_1 \in [K_1, K_{-1}] \) (the shaded are in the figure), the CRA is able to influence firms’ project choice at \( t = 1 \). With beliefs updated only after observing project returns, \( \hat{\alpha}^s_1 \), the firm chooses the HR project. In this case, the CRA increases the precision of ratings such that under updated beliefs, \( \alpha^s_1 \), the G-rated firms would choose the LR project if they could refinance their debt to lower their cost of capital. If credit is unavailable however, the firms would continue to choose HR projects. Therefore, the lack of credit is an additional factor of systematic risk that coupled with a business downturn leads to the simultaneous increase of firms risk. We use this mechanism to explain the dynamics of spreads on the tranches of a collateralized debt obligation.
4 Securitized Debt

In this section we apply the model to price the tranches of a collateralized debt obligation (CDO). The pricing of the tranches of a synthetic CDO with a large homogeneous collateral pool is similar to that in Coval, Jurek, and Stafford (2009a) and Gibson (2005). We assume that at \( t = 0 \), the collateral pool consists of a large number of callable two-period bonds issued by \( G \)-rated firms, each with capital \( K_0 \). The project choices by these firms at \( t = 0 \), and the realized returns on these projects implies that at \( t = 1 \) their capital stocks differ, as do their leverage ratios. Moreover, the rating precision at \( t = 1 \) depends on firms’ capital, contributing to an additional dispersion in investors’ beliefs about these firms. As before, firms’ project returns bear purely idiosyncratic risk conditional on the state of the economy. At \( t = 1 \) if credit is available the firms decide whether to refinance their debt. If a firm refinances its bond, the newly issued bond replaces the old bond in the pool. Then the firms again choose a project and end up with capital \( K_2 \) at \( t = 2 \).

Since at \( t = 1 \) the pool is heterogeneous we cannot obtain the distribution of losses in a pool at \( t = 2 \) in closed form as in Gibson (2005). Instead, to estimate the losses and determine the tranche spreads at \( t = 1 \) we resort to a simulation technique. First, we simulate the levels of capital of each firm in the pool at \( t = 1 \). In doing so, we fix state \( \hat{s} \) at \( t = 0 \) and form a pool of \( N \) firms with capital \( K_0 \) and randomly assigned type such that probability of the good type is \( \alpha_0 \). For each firm in the pool randomly determine the level of capital at \( t = 1 \) according to equation \( K^i_1 = K_0 r^i \), where random return \( r^i \) is drawn from the log-normal distributions with parameters \((\mu^s_P, \sigma^s_P)\) with \( P \) project choice of the good firms at \( t = 0 \) and \( s \) state at \( t = 1 \) for a good firm and \((\mu^s_{HR}, \sigma^s_{HR})\) for a bad firm.

Second, for each state \( s \) at \( t = 1 \) simulate the distribution of losses in the pool and calculate average losses \( L[A^L, A^U] \) on each CDO tranche with attachment points \( A^L \) and \( A^U \). In particular, we conduct \( T \) Monte-Carlo simulations and in each trial:

1. Given state \( s \) at \( t = 1 \) and transition probabilities \( \lambda^Q_{ss'} \), randomly choose state \( s' \) at \( t = 2 \).

2. For each firm in the pool randomly determine the level of capital at \( t = 2 \) according to equation \( K^i_2 = K^i_1 r^i \), where random return \( r^i \) is drawn from the log-normal distributions
with parameters \((\mu^s_p, \sigma^s_p)\) with \(P\) project choice of the good firms at \(t = 1\) and \(s'\) state at \(t = 2\) for a good firm and \((\mu^s_{HR}, \sigma^s_{HR})\) for a bad firm.

3. Calculate the value \(D^i_2(K_2)\) of \(i\)th bond according to (11) and the total payoff of the portfolio of \(N\) bonds

\[
D^*_2 = \frac{\sum_{i=1}^{N} D^i_2(K_2)}{\sum_{i=1}^{N} F^i_{s2}},
\]

where \(F^i_{s2}\) is equal \(F^i_{02}\) if the \(i\)th bond has not been refinanced or \(F^i_{12}\) otherwise.

4. Calculate expected loss on each tranche with lower and upper attachment points \(A^L\) and \(A^U\)

\[
L[A^L, A^U] = \max(L^*_2 - A^L, 0) - \max(L^*_2 - A^U, 0),
\]

where \(L^*_2 = 1 - D^*_2\).

Finally, given states at \(t = 0\) and \(t = 1\) we use the average losses on each tranche to calculate the spreads for each tranche using equation

\[
S^*_ss[A^L, A^U] = \frac{A^U - A^L}{A^U - A^L - L[A^L, A^U]} - 1.
\]

5 Empirical Analysis

In this section, we structurally estimate our model and evaluate its implications for the pricing of CDO tranches. Our empirical estimation is implemented in two stages. At the first stage we use standard maximum likelihood of regime switching models (see Hamilton (1994)) to estimate the cycles in credit availability and macroeconomic growth. The regimes are observed by the agents in the model, but are unobserved by the econometrician. In the second stage, we use the simulation method of moments (SMM) to estimate the parameters of firms’ projects that fit tranche spreads.

5.1 First Stage Maximum Likelihood Estimation of Regime Switching Model

The specification of the regime model is at the beginning of Section 2. Macro cycles are identified as regimes of real GDP growth (states B and R), while credit cycles are identified as
regimes in the ratio of credit growth at nonfinancial companies to nominal GDP (A and N). We then form the four composite states (BA), (R,A), (BN), and (R,N). The specification has homoskedastic fundamentals, so that the volatility of each process is the same in each regime. We maximize the likelihood of the econometrician observing these four composite regimes. It is useful to note, that we estimate the model from 1951 to 2004, before the start of the CDO tranche data. Using these estimates, we filter the data to provide the econometrician’s filtered probability of the underlying states in-sample (1951 – 2004:Q2) and then out-of-sample (2004:Q3 – 2014). By doing so, we attempt to mitigate over-fitting of tranche spreads in the second subsample. The time series of the econometrician’s filtered probabilities are denoted as \( \{ \omega^{mc}(t) \} \).

Parameter estimates of the model are in Table 2. As seen in the top panel, the ratio of credit growth to GDP is about 2.5 times as high in A states relative to N states, although even the latter is positive. This is consistent with our model in which firms can obtain credit with some probability even in N states. Real GDP growth is about 4.5 percent (at an annual rate) in B states, and shrinks at nearly 1 percent in R states. GDP growth is significantly more volatile than credit growth. The quarterly transition matrix (and its standard errors) under the objective measure are in the two subsequent panels. We will discuss the risk-adjusted transition matrix (under the Q-measure) in a subsequent subsection. As in several other estimates of growth regimes, booms are far more persistent than recessions. In addition, booms are more persistent in credit availability states. Indeed, the RN state is the least persistent.

The econometrician’s filtered probabilities of the four composite regimes are in Figure 4. As seen, each of the four regime probabilities become quite likely in different stages of the cycles. Quite notably, the probabilities of BA regimes increase significantly in the middle of most NBER recessions (shaded areas) in the sample. However, the probabilities of N (credit unavailability) regimes, remain quite high even after the end of recessions. Therefore, credit availability lags GDP growth, and a simple linear regression of credit growth on lagged GDP growth verifies this intuition.

\[
\text{Ratio of Credit Growth}(t)/\text{GDP}(t) = 0.219 + 0.451 \text{ GDP Growth}(t - 4) + \epsilon(t) \quad (16)
\]

\[
= \begin{bmatrix} 1.431 \\ 3.306 \end{bmatrix} \quad (17)
\]
where t-stats adjusted for autocorrelation and heteroskedasticity are in parenthesis.

Figure 5 shows that the expected growth rates from the model, fit the data quite well. In fact, the regression of each of the realized data series on its expected value calculated using the regime parameters and the filtered probabilities explains close to 57 percent of the variation in each series. The plots also show that expected GDP growth troughs in recessions, while expected credit growth troughs 2 to 4 quarters after the end of each recession. This was specially true for the last three recessions.

While we have a reduced form specification of macro and credit regimes, our estimates are consistent with the view that credit availability shrinks at the onset of weak growth, and persists for several quarters even after growth resumes, perhaps because lenders become more cautious.

5.2 Second Stage SMM Estimation of Firms’ Project Return Parameters

We now provide a description of the SMM estimation of the parameters of firms’ projects and the risk adjustment demanded by investors, which are estimated at the second stage. These parameters are chosen to match the time series of spreads using the econometrician’s filtered probabilities of the states (regimes) at the first stage. We recall, that the probabilities are out-of-sample from the first stage estimation. To fit spreads, we use the pricing formulae of tranche and the entire CDX spread developed in Section 4. To implement the 3-period model, we use time periods of unequal length. The physical time between periods 0 and 1, is 1-year, and the time between periods 1 and 2, is 4-years. Recall that the bond is callable after the first period, which is similar to actual callability restrictions on bonds, which can be called for only a fraction of their maturities.

Using the filtered probabilities, we have

\[ S_t[A^L, A^U] = \sum_{s \in S} \frac{\lambda^Q_{ss} \omega^s_t}{\sum_{r \in S} \lambda^Q_{rs}} S_{t-1}[A^L, A^U]. \]  

As in the Section 4, tranche spreads at \( t \) depend on the face value of debt issued at \( t - 1 \), and hence the expected spread at \( t \) depends not only on the probabilities of states at \( t \), but in addition, states at \( t - 1 \).
The parameters that we need to estimate are a) \( \mu_p \) and \( \sigma_p \), for \( p \in \{LR, HR\} \), and \( s \in \{B, R\} \), resulting in 8 parameters. We also estimate \( \beta_1 \) and \( \beta_2 \), which are the parameters to risk-adjust the transition probability matrix, overall resulting in 10 parameters.

For the SMM procedure we use the time series of the five spreads (one for the full pool, CDX, and four tranches). We also calculate the conditional volatility of spreads at each date using the filtered probabilities, \( \omega_{mc}(t) \), and match these to the unconditional sample volatility of each spread. In addition, we calculate the P-measure probability conditional probability of default, using the simulated beliefs of investors of each firm being good states. The average of this time series is used to match the historical 4-year default probability of BBB-rated firms by S&P. Finally, we target the endogenously determined leverage ratio in the model at each date, to match the unconditional average of leverage of BBB-rated firms in the data. This gives us 11 moments to match, overall leading to an overidentified identified SMM estimator.

The parameter estimates are given in Table 3. As seen, both type of projects are riskier in recession states. In addition, HR projects have higher risk and lower returns than LR projects in each state.

5.3 Risk-Adjustment Parameters

The signs of the risk-adjustment for the growth and credit growth transitions in Table 3 are quite compelling. As in common parlance, the price of risk of a shock is positive (negative) if it over (under) weights the transition probability from a good to a bad state for investors. For our estimated parameters, we find \( \beta_1 \), the adjustment for the growth transition probability is positive, as is consistent with several other empirical studies. Quite interestingly, our estimate of \( \beta_2 \), the credit growth transition, is negative. Above, we showed that credit growth remains strong at the start of a recession, but then weakens, and remains weak after the end of the recession. Therefore, credit growth shocks have a slightly countercyclical property, and hence has a negative price of risk. The overall risk-adjustment across the composite states in our model does deliver us the increase in credit spreads, which measure expected default losses under the Q-measure, to match the historical spread level.
5.4 The Credit Spreads Puzzle, Spread Dynamics, and the Convexity Effect

Using the parameters estimated from the SMM procedure, we calculated the spreads for each of the tranches in period $t = 1$ of the model for each state at $t = 0$. The state at $t = 0$ is relevant for the spreads at $t = 1$, since it determines the proportion of firms choosing HR projects, and hence the face values of debt. These implied spreads are in Table 4. As seen senior spreads $S(15, 100)$ are zero in credit availability (A) states, while equity tranche spreads are close to their values in credit unavailability (N) states. In the model, during N states, several firms cannot refinance their existing debt, and hence, they choose HR projects. Therefore an increase in risk of some firms (relative to A states) implies that the chance of the equity tranches experiencing significant losses increases. But, since all firms do not increase their risk, the chance of all of them defaulting, an event that triggers losses in the senior tranche, does not increase. Instead, the spreads for senior tranches, are higher in low growth (R) states, where all firms’ volatility increases. This differential impact on senior and junior tranches helps our model match the different dynamics of these tranches.

Using these state-dependent spreads, we calculate the fitted spreads at each date, which are shown in Figure 6. Due to high risk-adjusted transition probabilities, the model is able to provide average spread levels fairly close to their historical averages, even as we match the objective-measure average default probability. As in the credit spreads puzzle literature, this happens, because more defaults happen in recessions, which have boosted transition probabilities under the Q-measure. In particular, the model’s spreads rose close to their historical values for junior tranches in the great recession, but fell a bit short for senior tranches. Also, significantly, the model’s senior tranche spreads, fell once economic growth picked up at the end of the recession, but the equity tranche in particular remained at high levels until nearly 2010, when credit growth resumed. This is in line with our motivating regressions in the introduction, where a much larger proportion of the equity tranche is explained by credit growth rather than economic growth, while the reverse is true for the senior tranche. Overall, the model-fitted spreads explain between 33 percent (equity tranche $S(0, 3)$) and 58 percent ($S(7, 15)$ tranche), with better for senior tranches.
One of the key aspects of our model is the endogenously generated convexity effect of credit spreads. As was pointed out by David (2008), essentially in structural form models of credit risk (such as this one), credit spreads are convex function of firms’ asset values (capital stocks). Due to heterogeneity in firms’ capital accumulation, spreads for firms with low realized capital rise more dramatically, then for the fall of spreads of firms that have high realized capital. The greater the dispersion in capital stocks across firms, the greater is the difference in average spreads across firms, and the spread calculated for a representative firm with an average capital stock. In the model, heterogeneity increases in low growth states, but also to some extent when credit is unavailable. Therefore spreads increase in such states. The convexity effect not only implies an increase in the average spread generated by the model, but also the dynamics of spreads, as spreads increases in states with higher dispersion, which endogenously varies as the economy transitions through the macro and credit states.

In Figure 6, in addition to the historical and model-fitted spreads, we plot the model-fitted spread without the convexity effect, by endowing each firm with the average capital stock of firms in the prevailing state. Our results are quite dramatic. For the entire pool, which is the CDX spread, the convexity effect accounts for between a third and half of the model spread, but for the junior tranches, the convexity effect is much larger. For example, for the equity tranche, the spread is almost negligible when we do not use the convexity effect, but close to its historical levels, once we use it. The convexity effect is smaller for senior tranches. Why is that? Because, the senior tranche gets hit only when all firms have low capital stocks, but in this case, they have lower dispersion in capital stocks.

As mentioned above, the convexity effect arises endogenously in our model. In particular, as the CRA changes the precision of its rating over time, it affects the dispersion in borrowing costs across firms, which in turn affects their project choices, and the dispersion in their capital stocks. This is a feature not present in prior work on the convexity effect, such as in David (2008).
6 Conclusion

In this paper, we provide a new model to show how imperfect credit ratings and occurrence of credit crunches can create catastrophic risk observed in the prices of CDO tranches. There are three crucial ingredients in our model. First, we endogenize firms’ risk-taking using the asset substitution mechanism. In particular, the firms choose the riskiness of their projects based on the amount of debt that they need to service. Second, a credit rating agency changes the intensity of the investigation of firms’ credit quality to maximize the proportion of firms with high credit ratings. Finally, the credit shortage can trigger firms’ risky behaviour if they are unable to refinance their debt under a more precise rating standard. This increases the risk of senior tranches of structured finance products.

We structurally estimate the parameters of our dynamic Bayesian persuasion model and show that it can shed light on the puzzling phenomenon that senior tranche spreads are relatively more exposed to growth shocks, while junior spreads are more exposed to credit availability shocks. In particular, refinancing of existing debt may not be possible during a credit crunch, and hence the resulting high risk strategy for some firms in such periods implies that junior tranches, get seriously impacted. In contrast senior tranches get affected by growth shocks, which increase the risk of all firms’ projects. A crucial aspect of our model is that an endogenously generated “convexity effect”, in large part due to the time varying precision of credit ratings, is much more important in understanding CDO tranche spreads than the spread on the entire pool of firms, the subject of past studies.

Data Appendix

We obtain monthly time series of tranche spreads on synthetic CDOs based on the DJ CDX North American Investment Grade Index (CDX.NA.IG). This index consists of an equally weighted portfolio of 125 credit default swap (CDS) contracts on US firms with investment grade debt. Our sample covers the eleven year period from September 2004 to October 2014. The data from September 2007 to October 2014 is provided by Bloomberg (CMA New York). The data from September 2004 to August 2007 is from Coval, Jurek, and Stafford (2009a).
The CDX indices roll every six months. In particular, on September 20 and March 20 new series of the index with updated constituents are introduced. After a new series is created, the previous series continue trading though liquidity is usually concentrated on the on-the-run series. An exception is series 9 introduced in September 2007 and traded till the end of 2012 together with less liquid on-the-run series. The CDX indices have 3, 5, 7 and 10 year tenors. We use 5 year CDX indices which are most liquid for most series.

We build our sample from on-the-run series except period from March 2008 to September 2010 where we use most liquid series 9. Before series 15 introduced in September 2010 the CDX index has been traded with tranches 0-3%, 3-7%, 7-10%, 10-15%, 15-30% and 30-100%. Starting from series 15 and onward, only odd series of the index are traded with tranches and the structure of tranches changes to 0-3%, 3-7%, 7-15% and 15-100%.

We focus our analysis on the equity and the most senior tranches. Since the equity 0-3% tranche is quoted as an upfront payment, we calculate the par spread using the formula $S_{0-3\%} = 500\text{ b.p. } + U/D$ where $U$ is the upfront fees and $D$ is the time to maturity of the tranche. While earlier series (before 15) have tranches 15-30% and 30-100%, there is only one tranche 15-100% for later series. To make the series consistent we create a tranche 15-100% for earlier series as the sum of tranches 15-30% and 30-100%.

We obtain credit growth at nonfinancial corporate businesses from the Federal Reserve Board’s flow of funds accounts (series FA104104005.Q), and nominal and real GDP from the St. Louis Fed FRED database.

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6The tickers of the tranches of series 9 are CT753589 Curncy, CT753593 Curncy, CT753597 Curncy, CT753601 Curncy, CT753605 Curncy, CT753609 Curncy.

7The tickers of the tranches of series 15, 17, 19 and 21 are CY071225 Curncy, CY071229 Curncy, CY071233 Curncy, CY071237 Curncy, CY087579 Curncy, CY087583 Curncy, CY087587 Curncy, CY087591 Curncy, CY125375 Curncy, CY125380 Curncy, CY125385 Curncy, CY125390 Curncy, CY181667 Curncy, CY181672 Curncy, CY181677 Curncy and CY181682 Curncy.
Appendix A

Proof of Lemma 1  The value of the equity is the expected value of the firm after debt repayment under limited liability, i.e.

\[ E_1(K_1) = \sum_{s' \in S} \lambda_{s,s'} \int_{0}^{\infty} (r K_1 - F)^+ f(r|\mu_{s'}, \sigma_{s'}) dr \]

\[ = \sum_{s' \in S} \lambda_{s,s'} \int_{F_0/K_1}^{\infty} (r K_1 - F) f(r|\mu_{s'}, \sigma_{s'}) dr \]

\[ = \sum_{s' \in S} \lambda_{s,s'} \left[ K_1 \mathbb{E}[\tilde{r}|\tilde{r} > F/K_1, \mu_{s'}, \sigma_{s'}] - F \mathbb{P}[\tilde{r} > F_0/K_1|\mu_{s'}, \sigma_{s'}] \right], \]

where \( f(r|\mu, \sigma) \) is the PDF of the log normal distribution with parameters \( \mu \) and \( \sigma \). Now using standard results on the log normal distribution implies that (4) holds. ■

Proof of Lemma 2  The CRA’s problem is

\[ \max_{\delta_1, \delta_2} \mathbb{P}[G] \quad (19) \]

\[ \text{such that } \mathbb{P}[g|G] \geq \theta', \quad (20) \]

where \( \theta' \) is the target level of beliefs. By the law of total probability

\[ \mathbb{P}[G] = \mathbb{P}[G|g]\mathbb{P}[g] + \mathbb{P}[G|b]\mathbb{P}[b] = \theta \delta_1 + (1 - \theta) \delta_2. \quad (21) \]

Given \( \theta \), to maximize unconditional probability \( \mathbb{P}[G] \) the CRA chooses probabilities \( \delta_1 \) and \( \delta_2 \) as large as possible (but not greater than one). The optimal solution follows from the fact that these variables are related by (20), i.e.

\[ \mathbb{P}[g|G] = \frac{\mathbb{P}[G|g]\mathbb{P}[g]}{\mathbb{P}[G|g]\mathbb{P}[g] + \mathbb{P}[G|b]\mathbb{P}[b]} = \frac{\theta \delta_1}{\theta \delta_1 + (1 - \theta) \delta_2} \geq \theta', \quad (22) \]

or, equivalently,

\[ \delta_2 \leq \frac{\theta(1 - \theta')}{\theta'(1 - \theta)} \delta_1. \quad (23) \]

Conditions (23), \( \delta_1 \leq 1 \) and \( \delta_2 \leq 1 \) imply that maximum values of \( \delta_1 \) and \( \delta_2 \) are given by (7) and (8). ■
Proof of Lemma 3  The value of the debt is the expected value of repayment, i.e.

\[ D_1(K_1) = \sum_{s' \in S} \lambda_{s,s'}^{Q} \int_{0}^{\infty} D_2(rK_1) f(r|\mu_{p}^{s'}, \sigma_{p}^{s'}) dr, \tag{24} \]

where \( f(r|\mu, \sigma) \) is the PDF of the log normal distribution with parameters \( \mu \) and \( \sigma \). Since

\[ D_2(K_2) = \begin{cases} F_{02}, & \text{if } K_2 \geq F_{02} \\ (1 - \delta)K_2, & \text{if } K_2 < F_{02}, \end{cases} \tag{25} \]

the expected repayment can be written as

\[
\begin{aligned}
D_1(K_1) &= \sum_{s' \in S} \lambda_{s,s'}^{Q} \left[ \int_{0}^{F_{02}/K_1} (1 - \delta)K_1 f(r|\mu_{p}^{s'}, \sigma_{p}^{s'}) dr + \int_{F_{02}/K_1}^{\infty} F_{02} f(r|\mu_{p}^{s'}, \sigma_{p}^{s'}) dr \right] \\
&= \sum_{s' \in S} \lambda_{s,s'}^{Q} \left[ (1 - \delta)K_1 \mathbb{E}[\tilde{r}|\tilde{r} \leq F_{02}/K_1, \mu_{p}^{s'}, \sigma_{p}^{s'}] \mathbb{P}[\tilde{r} \leq F_{02}/K_1|\mu_{p}^{s'}, \sigma_{p}^{s'}] \\
&\quad + F_{02} \mathbb{P}[\tilde{r} > F_{02}/K_1|\mu_{p}^{s'}, \sigma_{p}^{s'}] \right], \tag{26} \\
\end{aligned}
\]

where \( \tilde{r} \) is log normally distributed random variable. Using the fact that the expected value of the truncated from above log normal distribution is

\[ \mathbb{E}[\tilde{x}|\tilde{x} \leq u] = \exp(\mu + \sigma^2/2) \Phi(\bar{u} - \sigma)/\Phi(\bar{u}), \tag{27} \]

where \( \bar{u} = (\ln u - \mu)/\sigma \) and \( \Phi(x) \) is the standard normal CDF, we have the result. \( \blacksquare \)
Table 1: What Explains CDO Tranche Spreads?

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<td>[-4.73]</td>
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<td>6</td>
<td>1779.44</td>
<td>-205.87</td>
<td>-604.85</td>
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<td>[17.01]</td>
<td>[-3.58]</td>
<td>[-4.42]</td>
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<tr>
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<td>7</td>
<td>627.11</td>
<td>-362.28</td>
<td>0.362</td>
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<tr>
<td></td>
<td>[5.48]</td>
<td>[-3.04]</td>
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<td>8</td>
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<td>[114.18]</td>
<td>[126.75]</td>
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<td>9</td>
<td>777.69</td>
<td>-300.23</td>
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<td>Spread (7-15)</td>
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<td>406.83</td>
<td>-343.59</td>
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<td></td>
<td>[4.53]</td>
<td>[-3.32]</td>
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<tr>
<td>11</td>
<td>410.68</td>
<td>-333.43</td>
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<td>[3.57]</td>
<td>[-2.60]</td>
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<td>12</td>
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<td>-272.17</td>
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<td>Spread (15-100)</td>
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<td>-49.61</td>
<td>-25.26</td>
<td>0.653</td>
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<tr>
<td></td>
<td>[-5.28]</td>
<td>[-3.17]</td>
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</tbody>
</table>

Tranche spreads are on the Dow Jones North American Investment Grade Index, which are reported by Credit Market Analysis (CMA) and obtained from Bloomberg (see Data Appendix for construction of our time series). CDX represents the full CDO. Spread (AL,AU) represents the spread on a tranche with loss attachment points AL and AU in percentage points. For example, the “senior” spread represents the 15 to 100 percent loss attachment points, while the “equity” tranche represents the 0 to 3 loss attachment points. We report the coefficients of the fitted regression:

\[
\text{Tranche Spread}(t) = \alpha + \beta_1 \text{Real GDP}^{\text{Growth}} + \beta_2 \text{Credit Growth}(t)/\text{GDP}(t) + \epsilon(t)
\]

for alternative tranches T-statistics are in parenthesis and are adjusted by White’s procedure for heteroskedasticity.
Table 2: Maximum Likelihood Estimates of 4-Regime Markov Switching Model for Ratio of Credit Growth at Nonfinancial Firms to GDP and Real GDP Growth

<table>
<thead>
<tr>
<th>Ratio of Credit Growth to GDP (%)</th>
<th>$\mu_1^c$</th>
<th>$\mu_2^c$</th>
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<tbody>
<tr>
<td></td>
<td>0.901</td>
<td>0.366</td>
</tr>
<tr>
<td></td>
<td>(0.185)</td>
<td>(0.005)</td>
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</table>

<table>
<thead>
<tr>
<th>Quarterly Real GDP Growth (%)</th>
<th>$\mu_1^g$</th>
<th>$\mu_2^g$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1.109</td>
<td>-0.237</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.002)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volatilities (%)</th>
<th>$\sigma_g$</th>
<th>$\sigma_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.251</td>
<td>0.755</td>
</tr>
<tr>
<td></td>
<td>(7.432)</td>
<td>(0.019)</td>
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</table>

Standard errors are in parenthesis.

<table>
<thead>
<tr>
<th>Quarterly Transition Probability Matrix (Estimates)</th>
</tr>
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<tr>
<td>(BA)</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>(BA)</td>
</tr>
<tr>
<td>(RA)</td>
</tr>
<tr>
<td>(BN)</td>
</tr>
<tr>
<td>(RN)</td>
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<table>
<thead>
<tr>
<th>Quarterly Transition Probability Matrix (Asymptotic Standard Errors)</th>
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<tbody>
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</tr>
<tr>
<td>(BA)</td>
</tr>
<tr>
<td>(RA)</td>
</tr>
<tr>
<td>(BN)</td>
</tr>
<tr>
<td>(RN)</td>
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</table>

Log Likelihood = -401.725
Table 3: Second Stage SMM Estimation of Firms’ Project and Risk Adjustment Parameters

<table>
<thead>
<tr>
<th>Firms’ Project Parameters</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{BR}$</td>
<td>0.132</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{BR}^2$</td>
<td>0.019</td>
<td></td>
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<td></td>
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<tr>
<td>$\mu_{LR}$</td>
<td>0.047</td>
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<tr>
<td>$\sigma_{LR}^2$</td>
<td>0.035</td>
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<td></td>
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<tr>
<td>$\mu_{HR}$</td>
<td>0.022</td>
<td></td>
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<tr>
<td>$\sigma_{HR}^2$</td>
<td>0.036</td>
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<tr>
<td>$\mu_{HR}$</td>
<td>0.002</td>
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<tr>
<td>$\sigma_{HR}^2$</td>
<td>0.152</td>
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<table>
<thead>
<tr>
<th>Risk Adjustment Parameters</th>
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</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>0.129</td>
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<tr>
<td>$\beta_2$</td>
<td>-0.220</td>
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J-Statistic = 0.988

Table 4: Implied Spreads (In Basis Points) From SMM Parameter Estimates

<table>
<thead>
<tr>
<th></th>
<th>CDX</th>
<th>S(0,3)</th>
<th>S(3,7)</th>
<th>S(7,15)</th>
<th>S(15,100)</th>
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</thead>
<tbody>
<tr>
<td>State at t=0 is (BA)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>(BA)</td>
<td>27</td>
<td>559</td>
<td>258</td>
<td>36</td>
<td>0</td>
</tr>
<tr>
<td>(RA)</td>
<td>290</td>
<td>1344</td>
<td>1067</td>
<td>1067</td>
<td>173</td>
</tr>
<tr>
<td>(BN)</td>
<td>37</td>
<td>1031</td>
<td>278</td>
<td>29</td>
<td>0</td>
</tr>
<tr>
<td>(RN)</td>
<td>243</td>
<td>5729</td>
<td>873</td>
<td>574</td>
<td>119</td>
</tr>
<tr>
<td>State at t=0 is (BN)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(BA)</td>
<td>27</td>
<td>559</td>
<td>258</td>
<td>36</td>
<td>0</td>
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<tr>
<td>(RA)</td>
<td>234</td>
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<td>1067</td>
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<td>1031</td>
<td>278</td>
<td>29</td>
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<tr>
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<td>1422</td>
<td>574</td>
<td>574</td>
<td>74</td>
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<tr>
<td>State at t=0 is (RA)</td>
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<td></td>
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<td></td>
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<tr>
<td>(BA)</td>
<td>27</td>
<td>559</td>
<td>258</td>
<td>36</td>
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<tr>
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<td>243</td>
<td>5729</td>
<td>873</td>
<td>574</td>
<td>119</td>
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<tr>
<td>State at t=0 is (RN)</td>
<td></td>
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<td></td>
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<tr>
<td>(BA)</td>
<td>27</td>
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<td>198</td>
<td>2529</td>
<td>627</td>
<td>574</td>
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</table>
Figure 1: Tranche Spreads, Economic Growth, and Credit Availability

Tranche spreads are on the Dow Jones North American Investment Grade Index, which are reported by Credit Market Analysis (CMA) and obtained from Bloomberg (see Data Appendix for construction of our time series). The “senior” spread represents the 15 to 100 percent loss attachment points, while the “equity” tranche represents the 0 to 3 loss attachment points.
Good firms choose LR or HR and invests all its capital.

The firm issues a two-period bond with face value $F_{02}$ and call price $H$.

If the credit is available, the firm may refinance its debt, i.e. pay call price $H$ and issue a one-period bond with face value $F_{12}$ to finance the repayment.

If refinanced at $t = 1$ the firm repays $F_{12}$; otherwise the firm repays $F_{02}$.

The CRA produces ratings and moves investors’ belief from $\hat{\alpha}_0$ to $\alpha_0$.

Investors observe project outcome and update their beliefs to $\hat{\alpha}_1$. If necessary, the CRA adjusts ratings so that investors update their beliefs from $\hat{\alpha}_1$ to $\alpha_1$.

The firm issues a two-period bond with face value $F_{02}$ and call price $H$.

Good firms choose LR or HR and invests all its capital.

Good firms choose LR/HR and invests all its capital.
Figure 3: Belief Updating From Learning and Bayesian Persuasion

Parameters: $\tilde{r}_{LR} \sim LN(0.3, 0.3), \tilde{r}_{HR} \sim LN(0, 0.8), K_0 = 1, \tilde{D} = 0.5, \alpha_0 = 0.2.$
Figure 4: Probabilities of the States From Regime Switching Model (1950Q1 – 2014Q4)
Figure 5: Fundamentals: Data and Fitted From Regime Switching Model (1950:Q1 - 2014:Q4)

- Credit Growth to GDP (Data)
- Credit Growth to GDP (Model)

R-square = 56.8%

R-square = 57.8%
Figure 6: Model and Actual Spreads on Senior and Equity Tranches

- **CDX Spread (Entire Pool)**
  - R-square = 57.6%

- **Spread (0,3)**
  - R-square = 32.8%

- **Spread (3-7)**
  - R-square = 46.8%

- **Spread (7-15)**
  - R-square = 57.8%

- **Spread (15-100)**
  - R-square = 52.4%
References


