A Theory of Liquidity Spillover Between Bond and CDS Markets

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I build a search model of bond and CDS markets that features endogenous funding liquidity and interdependent bond and CDS market liquidity. I show that, in the long run, speculative CDS purchases attract greater funding liquidity into credit markets that then, due to search frictions, spills over and increases bond market liquidity. In the short-run, however, the aggregate capital invested in credit markets remains fixed and, as a result, speculative CDS purchases attract liquidity away from the bond market. The opposite short- and long-run effects help explain the observed effects of speculative CDS bans on liquidity of the underlying bond markets.

A large body of work explores, in the context of exchange traded assets, why financial derivatives exist and how they affect the underlying assets. A majority of asset classes, however, are traded over-the-counter (OTC). In the last decade, a significant research has emerged on liquidity and search frictions in OTC markets. Despite the insights from this literature, our understanding of the effects of derivatives is limited in the context of OTC

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1OTC markets include fixed income (corporate bonds, government bonds, municipal bonds, the interbank market), commodity, currency, mortgage and other asset backed securities, structured products, interest rate rate swaps and other derivatives on these underlying.
traded assets. In this paper, I shed light on how derivatives affect liquidity and prices of the underlying assets when both assets are traded OTC. I address this question with a specific application to sovereign bond and credit default swap (CDS) markets.\(^2\)

The controversy surrounding CDS during the recent European debt crisis culminated in bans on “naked” purchases of CDS. Naked CDS purchases refer to the purchase of CDS protection by an investor who does not actually own the underlying bonds. As quasi-natural experiments to the CDS market, these policies allow us to empirically identify how naked CDS trading affects the underlying bonds. Sambalaibat (2014) documents that permanent and temporary CDS bans had exactly opposite effects on bond market liquidity. When the EU voted in October 2011 to permanently ban naked CDS referencing EU countries, countries affected by the ban experienced a decrease in their bond market liquidity (measured by bond bid-ask spreads). When Germany temporarily banned naked CDS in May 2010, this pattern reversed: Bond market liquidity temporarily increased instead.

In this paper, I build a dynamic search model of bond and CDS markets and show that the opposite changes in bond market liquidity arise from slow moving capital. In the short-run, investors keep the aggregate capital invested in credit markets fixed and substitute only locally between bond and CDS markets. Consequently, as liquidity demanders, naked CDS buyers compete for and attract liquidity away from the bond market. In the long-run, however, investors reallocate their investment funds at a wider scale in and out of credit markets. Naked CDS buyers, as a result, attract greater funding liquidity into credit markets that, in turn, spills over and increases bond market liquidity because of search frictions.

Permanent and temporary CDS bans reverse the long- and short-run effects, respectively. When the ban is permanent, liquidity suppliers, who are forced to permanently exit the CDS market, pull out from the bond market also. The result is a decrease in bond market liquidity. But when the ban is only temporary, they temporarily substitute out of the CDS into the bond market. The effect is a temporary increase in bond market liquidity.

In the model, I capture the over-the-counter structure of bond and CDS markets with the search and bilateral bargaining mechanism of Duffie, Garleanu, and Pedersen (2005, 2007). A fraction of bond owners are hit by a liquidity shock that requires them to sell their bonds. Locating a buyer, however, involves search costs. When a seller finds a buyer, she accounts for the difficulty of locating another buyer and resorts to selling her bond at a

\(^2\)A buyer of a CDS protection pays a periodic fee until either the contract matures or a default (or a similar event) occurs. In return, the protection seller transfers the purchased amount of insurance in the event of default. The contract specifies the reference entity, the contract maturity date, the insurance amount, and the events that constitute a credit event.
discounted price. Thus, as in the standard search framework, search costs create an illiquidity discount in bond prices.

I study how CDS contracts affect the illiquidity discount in bond prices, bond bid-ask spreads, and bond trading volume by modeling two novel features. The first is the presence of CDS markets. CDSs are zero net supply derivative assets; bonds are fixed supply assets. Similar to bonds, trading CDS contracts involves search costs. CDSs exist in the model because they complete markets. CDSs and bonds pay off in different (default and non-default) states. Buying naked CDS allows short positions with respect to the underlying bonds that are otherwise not possible because investors cannot directly short bonds. This assumption captures a fundamental difference between bond and CDS markets: It is cheaper to short credit risk using the CDS market than through the bond market.

The second novel feature is endogenous funding liquidity. In particular, the entry decision of investors who prefer the capital intensive side of bond and CDS trades (hence, in the position of supplying liquidity) adjusts endogenously to the introduction and the shut down of the CDS market. The model thereby allows for distinct notions of funding liquidity and market liquidity, and both are endogenously determined. I refer to the effects when entry is fixed as a short-run effect and when entry is endogenous as a long-run effect.

In this environment, the short-run effect is intuitive, and similar results have been highlighted before in the literature. The long-run spillover effect, however, is a novel insight and works as follows. Allowing investors to short the underlying bonds by buying CDS attracts investors who want to take the other side of the trade and hold long positions. These are the investors in the position of supplying liquidity into either market by buying bonds (from investors looking to liquidate) or selling CDS to speculators. Because buying bonds and selling CDS are close substitutes and both involve search frictions, long investors face increasing returns to scale from searching simultaneously in both markets. Long investors, consequently, search and trade in both markets, which gives rise to the spillover effect.

This paper contributes to the literature in two important respects. First, it provides, to the best of my knowledge, the first theoretical framework of OTC trading in both the underlying and derivative markets. The model features endogenous funding liquidity and prices and liquidity that are jointly determined across bond and CDS markets. Thus, beyond explaining the ban effects, the structural framework can address questions that have been explored empirically so far and, consequently, with results limited by potential endogeneity problems. To disentangle the determinants of the CDS-bond basis, I show, in an extension, how exogenous changes in bond and CDS market liquidity and in funding liquidity affect the basis.
The CDS literature has so far focused on covered CDS, but the debate surrounding CDS has been about the speculative use of CDS (i.e., naked CDS purchases). Thus, my second contribution is to shed light on the effect of naked CDS purchases. Naked CDS purchases lie at the heart of what makes CDS a derivative instrument for two contractual features of CDS. First, in contrast to bonds, CDS is a standardized instrument. It is written on the universe of all bonds of the bond issuer, not on individual bonds. As a result, CDS is a popular instrument to trade the overall credit risk of the issuer while avoiding bond specific risk.\(^3\) Second, shorting bonds is inherently limited by the supply of bonds and, hence, relatively costly. The analogous supply constraint does not exist for CDS. Together, these two features make CDS a convenient instrument to short the underlying issuer to hedge long exposures correlated with the underlying risk. Both this type of CDS purchase and the CDS purchase for outright speculating and shorting are naked CDS purchases. Moreover, the fact that trade in European sovereign single-name contracts has significantly dried up since the naked CDS bans is a further evidence that naked CDS positions were a large and important part of the CDS market.\(^4\)

The results of my model offer three novel insights. First, they show that endogenizing the aggregate number of investors is particularly important when studying the effect of an additional market, a trading venue, or an instrument. In contrast, in existing theories of liquidity interaction between multiple markets, the aggregate number of traders is held fixed; as a result, introducing additional markets, by construction, fragments traders across multiple markets.

Second, search frictions in the CDS market are a key model ingredient for the spillover effect and, hence, an important part of the explanation for the empirical patterns in bond market liquidity. Previous theories on CDS are unsatisfactory because they rely on the assumption that CDS contracts are more liquid than bonds. In the data, Sambalaibat (2014) documents the opposite: For sovereign names, CDS bid-ask spreads are on average ten times larger than bond bid-ask spreads. In my model, CDS exists, not because it is more liquid, but because it allows investors to short the underlying bonds cheaply. In the absence of search frictions in the model, the existence of naked CDS buyers is redundant.

Third, my results are novel because existing notions of how CDS might affect the bond market cannot explain why different CDS bans would have

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\(^3\)In the context of sovereign CDS, the contrast between the heterogeneity of bonds and the standardization of CDS is even starker.

\(^4\)Between the EU’s introduction of the permanent ban in October 2011 and June 2013 (the end of my sample period), the total amount of CDS purchased declined by one third. Since then trades have dried up further according ISDA (2014). In December 2014, for example, DeutscheBank which used to be one of the biggest dealers in the CDS market exited the single-name CDS market entirely.
different effects. It is commonly believed that investors are more willing to buy and hold bonds if they can hedge them with CDS (that is, hold covered CDS positions). This suggests that covered CDS positions increase bond market liquidity. In an extension, I verify that this is the case.\textsuperscript{5} Trading the CDS-bond basis would have a similar effect on bond market liquidity.\textsuperscript{6} Both covered CDS positions and basis trades necessarily involve a long position (with respect to the underlying) in one market and a short position (again, with respect to the underlying) in the other market. In contrast, in my mechanism, the increase in bond market liquidity is due to traders seeking a long position (with respect to the underlying) in \textit{both} markets. As for naked CDS trading, a potential effect is that it increases liquidity of the CDS market itself; consequently, it \textit{indirectly} increases bond market liquidity by decreasing the cost of buying CDS as a hedge for the underlying bonds. All of these effects cannot on their own explain the opposite effects.

Finally, I highlight two policy implications. First, my results imply permanently banning naked CDS trading adversely affected bond market liquidity, depressed bond prices, and, thereby, increased sovereigns’ borrowing cost exactly when governments were trying to avert a liquidity dry-up and credit risk spiral. This result is particularly important in the context of a sovereign debt crisis. Second, my results show that temporary versus permanent and anticipated versus sudden regulations can have exactly opposite consequences.

The paper is organized as follows. Section 1 presents the model environment, and Section 2 derives the main theoretical results. Section 3 gives additional results. Specifically, Section 3.1 shows how exogenously changing market liquidity and funding liquidity—through their effect on the endogenous liquidity of both markets—affect prices and the CDS-bond basis. Section 3.2 extends the model to allow bond holders to buy CDS and hold covered CDS positions. Section 3.3 calibrates the model and numerically illustrates the marginal effects of covered and naked CDS positions. Section 4 discusses assumptions and possible extensions, and Section 5 concludes. Appendix F gives institutional details on bond and CDS markets and the bans. Proofs of all the propositions are in Appendix A.

\textsuperscript{5}I show, in an extension, that covered CDS positions alone do not give rise to the opposite long- and short-run effects and, hence, cannot explain the ban effects.

\textsuperscript{6}In a basis trade, investors trade on an arbitrage opportunity that arises if bond and CDS markets price the underlying credit risk differently. For example, if the CDS premium is too low relative to bond yields, the CDS market is underestimating default risk relative to what the bond yields suggest. A basis trading strategy would be to buy the underpriced bonds and hedge their default risk with the currently cheap CDS.
Related Literature

This paper belongs to the search literature of financial assets beginning with the seminal papers Duffie, Garleanu, and Pedersen (2005, 2007). My framework is closely related to the extensions of their environment to multiple assets by Vayanos and Wang (2007) and Weill (2008). In particular, I adopt the framework of Vayanos and Weill (2008) that sheds light on the on-the-run phenomenon of Treasury bonds. I contribute to this literature, first, by modeling OTC trading in derivatives in addition to trading in the underlying asset and, second, by endogenizing agents’ entry decision into the market for the underlying asset in response to introducing the derivative market.

Afonso (2011) and Lagos and Rocheteau (2009) endogenize, in a search framework, the entry decision of traders and market-makers but in a single market setting. My model differs by featuring both multiple markets and endogenous entry. It therefore sheds light on the rate of entry into one market as a result of introducing another market and on the mechanism through which traders substitute between different markets.

A search theoretic paper applied specifically to CDS markets is Atkeson, Eisfeldt, and Weill (2012). They study, in a static setting, how banks’ CDS exposure arises endogenously depending on their size and exposure to aggregate risk. In contrast, first, I focus on naked CDS and, second, I allow trade in both the bond and the CDS market (as opposed to just the CDS market). Therefore, my model can speak to the endogenous feedback from the CDS market into the underlying bond market.

A related literature is equilibrium asset pricing models with exogenous trading frictions (e.g., Amihud and Mendelson (1986), Acharya and Pedersen (2005)). Oehmke and Zawadowski (2013) explore how CDS affects bond prices in Amihud and Mendelson (1986) type framework with exogenous trading frictions. In contrast, my model features endogenous trading costs and, thereby, an endogenous interaction and spillover between the underlying and derivative markets.

Using reduced form approaches, a growing number of papers price liquidity risk in bond and CDS markets. Related empirical papers study the joint dynamics of bond and CDS spreads and the price discovery mechanism in the two markets. An insight from this literature is that the relative liquidity

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8Studies of the relative price discovery in corporate bond and CDS include Blanco, Brennan, and Marsh (2005) and in sovereign bond and CDS: Fontana and Scheicher (2010), Arce, Mayordomo, and Peña (2012), Ammer and Cai (2011), Calice, Chen, and Williams (2013), and Delatte, Gex, and López-Villavicencio (2011). More specifically on the CDS-
in bond and CDS markets determines where the price discovery takes place and the extent of arbitrage opportunity between the two markets. My paper complements this literature by providing a tractable theoretical framework where liquidity and prices across the two markets are jointly determined.

My work is also related to the empirical literature on how CDS affects the issuer of the underlying debt. Ashcraft and Santos (2009) and Subrahmanyam, Tang, and Wang (2011) study the effect on firms’ cost of borrowing and credit risk, respectively. Massa and Zhang (2012) and Shim and Zhu (2010) document that CDS markets increased corporate bond market liquidity, and Das, Kalimipalli, and Nayak (2014) document the opposite. In contrast, my paper identifies the effect of naked CDS trading, not the CDS market in general.

For theory papers on CDS, Arping (2014) and Bolton and Oehmke (2011) formalize, in the context of corporate debt, the tradeoffs associated with the empty creditor problem and Sambalabat (2012) in the context of sovereign debt. Duffee and Zhou (2001) find that credit derivatives alleviate the lemons problem associated with banks having private information on their loans. Thompson (2007) and Parlour and Winton (2013) study the tradeoffs that banks face in selling versus insuring loans on their balance sheets. Thus, these papers have focused on issues surrounding covered CDS buyers who are directly exposed to the issuer’s default risk. I instead focus on how naked CDS buyers affect the issuer’s cost of borrowing through their effect on bond market liquidity and bond prices.

This paper also contributes to the theoretical literature that studies the endogenous cross-sectional distribution of liquidity and trade across markets. Information-based frameworks are Admati and Pfleiderer (1988), Pagano (1989), and Chowdhry and Nanda (1991). Search-theoretic frameworks are Vayanos and Wang (2007), Vayanos and Weill (2008), and Weill (2008). In these papers, traders endogenously concentrate in one market and trade in the other market can disappear. This result is a partial equilibrium effect bond basis, see, for example, Blanco, Brennan, and Marsh (2005) and Bai and Collin-Dufresne (2011). See Augustin (2014) for a survey of this literature.

Ashcraft and Santos (2009) find that CDS has beneficial effects on firms’ cost of borrowing for safer firms but adverse effects for riskier firms as banks may lose the incentive to monitor firms. Subrahmanyam, Tang, and Wang (2011) find CDS increases firms’ credit risk, which they attribute to protected creditors’ reluctance to restructure. Berndt and Gupta (2009) find that borrowers whose loans have been sold off underperform. I do not formally model the issuer’s borrowing cost in the primary debt markets. He and Milbradt (2014) provide a formal treatment of the feedback loop between credit risk, the issuer’s borrowing cost through the primary debt markets, and liquidity of the secondary bond markets.

Multiple markets can coexist under additional assumptions of heterogeneous agents and heterogeneous markets so that there is a “clienteles” effect. For example, Pagano (1989) shows that if markets differ in their fixed entry cost, then an equilibrium with multiple markets exists and has the following feature: The more liquid market has a larger fixed cost of entry and is also the market where only large traders (those needing a larger portfolio...
because, although the number of traders can vary in the cross-section, the aggregate number of traders is fixed. I get a similar result if the aggregate number of traders is fixed in my model. An important insight of my model is that if the aggregate number of traders is instead endogenous (that is, the endogeneity is on the extensive margin), the effect of an additional security is the opposite.

More broadly, this paper belongs to the literature on the effect of derivatives such as options and futures contracts. Theoretical frameworks that study the effect of derivatives on, specifically, liquidity of the underlying asset highlight the “migration” result similar to the above multiple-markets papers. Moreover, these papers are based on Kyle (1985) and Glosten and Milgrom (1985) frameworks where illiquidity arises from asymmetric information. The search framework of my paper is better suited for sovereign bond markets for two reasons. First, trade in sovereign bond markets is fragmented across heterogeneous bonds. Second, asymmetric information and insider trading are less severe with respect to governments than with respect to individual firms.

A majority of the literature on the effect of derivatives is empirical. The overwhelming empirical evidence is that derivatives generally increase liquidity of the underlying asset (Mayhew (2000) and references therein). But when the increase in the derivative activity is unexpected, the effect is instead a decrease in liquidity of the underlying (see, for example, Bessembinder and Seguin (1992)). Thus, we see effects similar to my model’s long- and short-run effects in different market contexts for derivatives and the underlying. The mechanism proposed in this paper shows that they arise because of slow moving capital.


See, for example, Chakravarty, Gulen, and Mayhew (2004) and the survey article, Mayhew (2000).
1 Model

Time is continuous and goes from zero to infinity. Agents are risk neutral, infinitely lived, and discount the future at a constant rate $r > 0$. A bond is an asset with supply $S$, pays coupon flow $\delta_b$, and trades at price $p_b$. Investors also trade a CDS contract in which a buyer of a CDS contract pays a premium flow $p_c$ to the seller and, in return, benefits from an expected insurance payment $\delta_c$. The bond coupon flow can be interpreted as an expected coupon flow: With intensity $\eta$, the bond defaults but otherwise pays a dollar of coupon. Hence, $\delta_b = (1 - \eta)\$1$. Similarly, $\delta_c$ can be interpreted as an expected insurance payment: A CDS contract pays out a dollar if there is a default on the coupon payment. Thus, $\delta_c = \eta\$1$. As in standard search models, default and credit risk are exogenous; the focus is instead on changes in asset prices through changes in asset liquidity.

Asset positions are denoted by $\theta_b$ for bond and $\theta_c$ for CDS. Let $\theta_i \in \{b,c\} = 1$ indicate a long position (exposed to risk), $\theta_i = 0$ no position, and $\theta_i = -1$ a short position with respect to underlying credit risk. CDS allows both long and short positions: $\theta_i \in \{-1, 0, 1\}$. A seller and a buyer of a CDS contract have long and short exposures to the underlying credit risk, respectively. I assume that bonds allow only long positions and that investors cannot directly short bonds: $\theta_b \in \{0, 1\}$. For simplicity, I restrict the net position to $|\theta_b + \theta_c| \leq 1$, which rules out holding a long position in each market simultaneously. When I refer to a long or short position, I will mean with respect to the underlying credit risk. Thus, a long position through the CDS market, for example, does not mean an investor has bought CDS but instead means she has sold CDS and, hence, is (long-) exposed to the underlying default risk.

Investors have heterogeneous valuations of asset cash flows. Holding $\theta_b$ units of the bond yields a utility flow $\theta_b (\delta_b + x^b_t) - |\theta_b|y$, and $\theta_c$ units of CDS yield $-\theta_c (\delta_c + x^c_t) - |\theta_c|y$. Here, $x^b_t \in \{-x_b < 0, 0, x_b\}$ and $x^c_t \in \{-x_{ch} < 0, 0, x_{cl} > 0\}$ are stochastic processes, and $y$ is a cost of risk bearing that is positive for both long and short positions. I define an agent with $\{x^b_t = x_b, x^c_t = -x_{ch}\}$ as a high-valuation agent, $\{x^b_t = 0, x^c_t = 0\}$ as an average-valuation agent, and $\{x^b_t = -x_b, x^c_t = x_{cl}\}$ as a low-valuation agent. This specification is shown in Table 1.
Table 1: Valuation of Bond and CDS Payments by High-, Average-, and Low-valuation Agents

Agents are heterogeneous in their valuations of bond and CDS cash flows. As shown in Bond Owner and CDS Seller columns, high-valuation agents derive a high utility from a long exposure to credit risk, but low-valuation agents derive a disutility from a long exposure. Conversely, low-valuation agents derive a higher utility from a short position (as shown in the CDS Buyer column), but high-valuation agents derive a disutility from a short position. As a result, in equilibrium, high-valuation agents hold long positions, low-valuation agents short credit risk, and average-valuation agents prefer to remain out of the markets.

<table>
<thead>
<tr>
<th></th>
<th>Long Credit Risk</th>
<th>Short Credit Risk</th>
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<tr>
<td></td>
<td>Bond Owner (\theta_b = 1)</td>
<td>CDS Seller (\theta_c = 1)</td>
</tr>
<tr>
<td>High</td>
<td>(\delta_b + x_b - y)</td>
<td>(- (\delta_c - x_{ch}) - y)</td>
</tr>
<tr>
<td>Ave</td>
<td>(\delta_b - y)</td>
<td>(-\delta_c - y)</td>
</tr>
<tr>
<td>Low</td>
<td>(\delta_b - x_b - y)</td>
<td>(- (\delta_c + x_{cl}) - y)</td>
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The parameters \(x_b\), \(x_{ch}\), and \(x_{cl}\) capture, in a reduced form, any incentives investors have for trading bond and CDS.\(^{14}\) One interpretation is that they are hedging benefits. In particular, investors have idiosyncratic endowments with heterogeneous correlations with the underlying bond cash flow. The endowment of a low-valuation agent is more correlated with the bond compared with the endowment of a high-valuation investor. Relative to a high-valuation agent then, a low-valuation agent derives less utility from a trade that makes her wealth even more correlated with the bond. Such trades are buying a bond \(\theta_b = 1\) or selling CDS \(\theta_c = 1\). Conversely, a trade that makes an investor’s position less correlated with the bond (such as buying CDS, \(\theta_c = -1\)) yields a greater utility to a low-valuation investor than to a high-valuation investor.\(^{15}\) Appendix C provides a micro-foundation and closed-form expressions for the hedging benefits \(x_b\), \(x_{ch}\), \(x_{cl}\), and \(y\) in an environment with risk-averse agents and risky assets. It shows that the hedging benefits increase with agents’ risk aversion, the bond cash-flow risk, and the correlation of agents’ endowment with the bond cash flow.

I keep \(x_b\), \(x_{ch}\), and \(x_{cl}\) general, but the results in the following sections hold even if they are the same. In the micro-foundation, the difference between the correlations of high and low investors’ endowment with the underlying bond determines the difference between \(x_{ch}\) and \(x_{cl}\). In absolute magnitudes, if the endowment of the low-valuation investor is more correlated with the

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\(^{14}\)For a similar setup, see Duffie, Garleanu, and Pedersen (2005) with two types of agents and Vayanos and Weill (2008) with three types of agents.

\(^{15}\)In particular, a low-valuation agent gets an extra disutility \(x_b\) from holding a bond \(\theta_b = 1\), while a high-valuation agent gets an extra utility \(x_b\). As for selling CDS \(\theta_c = 1\), a low-valuation agent experiences greater disutility paying out the insurance payments \(\delta_c - x_{ch} - y\) than a high-valuation agent \((\delta_c - x_{ch}) - y\). Buying CDS \(\theta_c = -1\) benefits a low-valuation agent more \((\delta_c + x_{cl}) - y\) than a high-valuation agent \((\delta_c - x_{ch}) - y\).
underlying credit risk, then \( x_{cl} > x_{ch} \); otherwise, \( x_c \leq x_b \). As for \( x_b \) versus \( x_{ch} \) and \( x_{cl} \), suppose \( x_{ch} = x_{cl} = x_c \). If the bond cash flow risk maps to a greater level of risk in the CDS cash flow, then \( x_c > x_b \); otherwise, \( x_c \leq x_b \).

CDS contracts, in reality, are bought (a) to hedge the underlying bonds, (b) to hedge risks correlated with the underlying (especially, in the context of sovereign CDS), or (c) to outright speculate and short. In the model and in the data, naked CDS purchases are the latter two. The micro-foundation in Appendix C is, however, an example of (b): using CDS to hedge risks correlated with the underlying.

Fixed flows of average-valuation agents \( F_h \) and \( F_l \) switch to high- and low-valuation agents, respectively. In turn, high- and low-valuation investors experience a liquidity shock and switch to an average-valuation with Poisson intensities \( \gamma_d \) and \( \gamma_u \), respectively. The constant flows and liquidity shocks generate trade in equilibrium.

**Assumption 1.** \( x_{ch} + x_{cl} > 2y > x_{ch} \).

Assumption 1 ensures that a CDS trade is profitable between high- and low-valuation investors but not between high- and average-valuation investors. If a low-valuation investor buys CDS from a high-valuation investor, the buyer’s flow surplus from the transaction is \( (\delta_c + x_{cl}) - y - p_c \), and the seller’s is \( p_c - (\delta_c - x_{ch}) - y \). The total surplus is then \( x_{ch} + x_{cl} - 2y \), which is positive from Assumption 1. If instead an average-valuation agent buys CDS from a high-valuation investor, the total surplus is \( x_{ch} - 2y \), which is negative from Assumption 1. Thus, the cost of the risk bearing parameter, \( y \), restricts the set of profitable trades between different agents and arises in a setting with more than two types of agents. For a CDS trade to be profitable, the difference in the involved parties’ valuations has to be large enough to offset the cost of risk bearing. The micro-foundation for the hedging benefits (Appendix C) gives a closed form expression for \( y \) and shows that it is an increasing function of the risk aversion parameter and the bond cash flow risk.

I restrict my analysis to parameter conditions such that, in the steady state, high-valuation investors are the marginal investors in the bond. This, together with the specification of hedging benefits and Assumption 1, ensures that, in the steady state, high- and low-valuation investors prefer to hold long and short positions, respectively, while average-valuation investors prefer to remain out of the markets.

I now explain how I endogenize funding liquidity. High-valuation investors are the investors in the position of supplying liquidity into the economy. They buy bonds from investors who need liquidity and are selling their assets. They sell CDS which, in reality, requires capital that has to be put aside as collateral. Thus, high-valuation investors trade as liquidity providers as they
prefer the capital intensive side of bond and CDS trades. I endogenize the entry rate (consequently, the aggregate number) of high-valuation investors. I refer to this as endogenous funding liquidity.\textsuperscript{16} A high-valuation agent enters to trade in bond and CDS markets if the expected value of doing so, denoted by $V_{h[0,0]}$, is at least greater than the value of her outside option, denoted by $O_h$. The outside option captures both the opportunity cost and direct costs of entering credit markets including, but not limited to, the cost of raising investment capital (both tangible and intangible such as expertise in credit markets). A fraction $\rho$ of investors choose to enter according to:\textsuperscript{17}

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\rho = \begin{cases} 
1 & V_{h[0,0]}(\rho) > O_h \\
[0, 1] & V_{h[0,0]}(\rho) = O_h \\
0 & V_{h[0,0]}(\rho) < O_h.
\end{cases}
$$

The total flow of high-valuation investors actually entering is then $\rho F_h$, and the steady state measures of high- and low-valuation investors are $\frac{\rho F_h}{7d}$ and $\frac{F_l}{7a}$, respectively. I focus on the range of parameters for which, in the steady state, $\frac{\rho F_h}{7d} > S + \frac{F_l}{7a}$ so that high valuation investors are the marginal long investors.

\subsection{The Bond and the CDS Market and Search Frictions}

Buyers and sellers in the bond market meet at a rate $M_b \equiv \lambda_b \mu_{bb} \mu_{bs}$, where $\lambda_b$ is the exogenous matching efficiency of the bond market, and $\mu_{bb}$ and $\mu_{bs}$ are the measures of bond buyers and sellers, respectively.\textsuperscript{18} Given the total meeting rate, buyers find a seller with intensity $\lambda_b \mu_{bs}$ per unit interval of time, and sellers find a buyer with intensity $\lambda_b \mu_{bb}$. The matching efficiency, $\lambda_b$, characterizes the extent of search frictions. The frictionless limit is characterized by $\lambda_b \to \infty$, in which case buyers and sellers find each other instantly. Once matched, a buyer and a seller bargain and split the surplus proportional to their respective bargaining powers: $\phi$ and $1 - \phi$.

Analogously, in the CDS market, CDS buyers find a seller with intensity $\lambda_c \mu_{ca}$, and sellers find a buyer with intensity $\lambda_c \mu_{cb}$, where $\mu_{cb}$ and $\mu_{cs}$ are the measures of CDS buyers and sellers, respectively. The total meeting rate is $M_c \equiv \lambda_c \mu_{cb} \mu_{cs}$.

\textsuperscript{16}See Rocheteau and Weill (2011) for a similar interpretation.

\textsuperscript{17}Afonso (2011) provides a more general setup in which there is a continuous distribution of agents with different outside values. My setup is a special case of this.

\textsuperscript{18}A general functional form for the matching functions is $M_b(\mu_{bb}, \mu_{bs}) = \lambda_b \mu_{bb}^{\alpha_{bb}} \mu_{bs}^{\alpha_{bs}}$ and $M_c(\mu_{cb}, \mu_{cs}) = \lambda_c \mu_{cb}^{\alpha_{cb}} \mu_{cs}^{\alpha_{cs}}$. Thus, I have implicitly assumed $\alpha_{bs} = \alpha_{bb} = \alpha_{cb} = \alpha_{cs} = 1$. Although constant returns to scale is standard in search models applied to labor markets, in the context of financial markets, the standard assumption is increasing returns to scale. Weill (2008) shows that comparative statics from a model with increasing returns to scale fit better the stylized facts regarding, for example, liquidity and asset supply.
1.2 Agent Types and Transitions

An agent of type \( \tau = i[\theta_b, \theta_c] \) is composed of his valuation type \( i \in \{h, a, l\} \) (high, average, low) and his asset position \([\theta_b, \theta_c]\). Feasible asset positions are a nonowner, \([\theta_b, \theta_c] = [0, 0]\); a bond owner, \([\theta_b, \theta_c] = [1, 0]\); a CDS seller, \([\theta_b, \theta_c] = [0, 1]\); and a naked CDS buyer, \([\theta_b, \theta_c] = [0, -1]\). To focus on the marginal effect of naked CDS positions, I rule out covered CDS positions \([\theta_b, \theta_c] = [1, -1]\). However, in Section 3.2, I relax this assumption. As shown in Table 2, the conjectured set of agent types is \( \mathcal{T} \equiv \{h[0, 0], l[0, 0], h[1, 0], a[1, 0], h[0, 1], a[0, 1], l[0, -1]\} \).

<table>
<thead>
<tr>
<th>Type</th>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>( h[1, 0] )</td>
<td>Has bought a bond</td>
</tr>
<tr>
<td>Bond owner</td>
<td>( h[0, 0] )</td>
<td>Fresh entrant; seeks a long position</td>
</tr>
<tr>
<td>Ave</td>
<td>( h[0, 1] )</td>
<td>Has sold CDS</td>
</tr>
<tr>
<td>Bond seller</td>
<td>( a[1, 0] )</td>
<td>Has bought a bond, seeks to sell it</td>
</tr>
<tr>
<td>Searcher CDS seller</td>
<td>( a[0, 1] )</td>
<td>Has sold CDS, seeks to unwind it</td>
</tr>
<tr>
<td>Low</td>
<td>( l[0, 0] )</td>
<td>Fresh entrant, seeks a short position</td>
</tr>
<tr>
<td>Naked CDS buyer</td>
<td>( l[0, -1] )</td>
<td>Has bought CDS</td>
</tr>
<tr>
<td>CDS holder</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Next, I describe for each agent type the conjectured optimal trading strategies and possible transitions (see Figure 1).

A high-valuation nonowner \( (h[0, 0]) \) seeks a long exposure to credit risk by buying a bond or selling CDS. I assume he can simultaneously search in both markets. Before he is even able to find a counterparty, he may switch to an average-valuation agent and exit the economy. But if he finds and trades with a bond-seller first, he becomes a high-valuation bond owner, \( h[1, 0] \). He is happy to hold this position until he is hit by a liquidity shock and becomes an average-valuation agent. He consequently becomes a bond seller \( (a[1, 0]) \) to liquidate his bond position and, upon finding a bond buyer, exits the economy.

If a high-valuation nonowner \( (h[0, 0]) \) instead bumps into a CDS buyer first and sells CDS (which occurs with intensity \( \lambda_c \mu_{cb} \)), he becomes a \( h[0, 1] \) type who is long-exposed to credit risk. He is happy with this position unless he switches to an average-valuation agent, \( a[0, 1] \). As an average-valuation agent, he seeks to unwind his position by searching for another CDS seller to take over his side of the contract. In practice, this is called assignment or
The figure shows the transitions between agent types. Flows of $\rho F_h$ and $F_l$ agents enter the economy as fresh high- and low-valuation investors. High- and low-valuation agents switch to an average-valuation with intensities $d$ and $u$, respectively. A trader seeking a long position ($h[0,0]$) finds a counterparty in the bond and CDS markets with intensities $\lambda_b \mu_{bs}$ and $\lambda_c \mu_{cb}$, respectively. A bond seller, $a[1,0]$, finds a buyer with intensity $\lambda_b \mu_{bs}$. A trader seeking a short position, $l[0,0]$, by buying CDS finds a counterparty with intensity $\lambda_c \mu_{cs}$.

<table>
<thead>
<tr>
<th>Asset Positions</th>
<th>High (h)</th>
<th>Average (a)</th>
<th>Low (l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0, 0]</td>
<td>$F_h$</td>
<td>$a[1,0]$</td>
<td>$l[0,0]$</td>
</tr>
<tr>
<td>Long Credit Risk</td>
<td>$[1,0]$</td>
<td>bond owner</td>
<td>naked CDS buyer</td>
</tr>
<tr>
<td></td>
<td>buys bond $\lambda_b \mu_{bs}$</td>
<td>sells bond $\lambda_b \mu_{bsa}$</td>
<td>buys CDS $\lambda_c \mu_{cs}$</td>
</tr>
<tr>
<td></td>
<td>$\gamma_d$</td>
<td>$\gamma_d$</td>
<td>$\gamma_u$</td>
</tr>
<tr>
<td></td>
<td>$h[0,0]$</td>
<td>exit</td>
<td>$l[0,0]$</td>
</tr>
<tr>
<td></td>
<td>long investor</td>
<td>counterparty cancels $\gamma_u$</td>
<td>CDS holder</td>
</tr>
</tbody>
</table>

For simplicity, I assume that the new seller that replaces him takes over the contract at a price originally contracted with the CDS buyer. A low-valuation nonowner ($l[0,0]$) seeks to short credit risk. She is the naked CDS buyer in the model. She searches to buy CDS and, with intensity $\lambda_c \mu_{cs}$, finds a counterparty and becomes a CDS holder ($l[0,-1]$). If she switches to an average-valuation agent, she terminates her contract. The termination forces her counterparty to search all over again if her counterparty is a high-valuation investor (i.e., he becomes $h[0,0]$ again). If her counterparty is instead an average-valuation investor, the counterparty simply exits the economy. Thus, CDS contracts are asymmetric: A CDS buyer can terminate, but a seller cannot terminate and instead has to find an investor willing to take over his side of the contract. I make this assumption to rule out and

\[ \text{novation. For simplicity, I assume that the new seller that replaces him takes over the contract at a price originally contracted with the CDS buyer.} \]

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\[ \text{novation. For simplicity, I assume that the new seller that replaces him takes over the contract at a price originally contracted with the CDS buyer.} \]
abstract from counterparty risk in CDS contracts.  

Given the conjectured equilibrium trading strategies, the measure of buyers and sellers in bond and CDS markets are \( \mu_{bb} = \mu_{h[0,0]} \), \( \mu_{bs} = \mu_{a[1,0]} \), \( \mu_{cs} = \mu_{h[0,0]} \), and \( \mu_{cb} = \mu_{l[0,0]} + \mu_{a[0,1]} \). In the steady state, for each type of investor, the inflow of agents has to equal the outflow so that the population measures are constant. For example, for the \( h[0,0] \) type,

\[
\rho F_h + \gamma_a \mu_{h[0,1]} = \gamma_d \mu_{h[0,0]} + (\lambda_b \mu_{bs} + \lambda_c \mu_{cb}) \mu_{h[0,0]},
\]

where the left-hand and right-hand sides reflect the inflows and outflows. Inflow-outflow equations for the other agents are analogously derived in Appendix A, Table 3.

1.3 Market Clearing

Bond market clearing imposes that the total measure of bond owners has to equal the bond supply:

\[
\mu_{h[1,0]} + \mu_{a[1,0]} = S.
\]

CDS market clearing requires that the total number of CDS contracts sold has to equal the number of CDS contracts purchased:

\[
\mu_{h[0,1]} + \mu_{a[0,1]} = \mu_{l[0,-1]}.
\]

1.4 Prices and Bargaining

Bond and CDS prices arise from bilateral bargaining between buyers and sellers. Let \( V_\tau \) denote the expected utility of investor type \( \tau \in T \). The buyer’s reservation value is the increase in his utility from buying a bond: \( V_{h[1,0]} - V_{h[0,0]} \). Similarly, the seller’s reservation value is \( V_{a[1,0]} \). The bond price is an average of the buyer and seller’s reservation value of the bond

\[
p_b = \phi V_{a[1,0]} + (1 - \phi) (V_{h[1,0]} - V_{h[0,0]})
\]

weighted by the bargaining power of the buyer (\( \phi \)) and the seller (\( 1 - \phi \)).

Thus, a buyer with a high bargaining power (\( \phi \)), for example, extracts a greater fraction of the total gains from trade by bargaining the price down.

\[20\] Alternatively, a buyer and a seller could be each required to post collateral to be seized upon default, as in actual CDS contracts. Typically, to prevent counterparty risk, the collateral posted by the seller is much larger than the collateral required from the buyer. My specification is a special case of this where the collateral required from the buyer is zero while the collateral required from the seller is infinity. It is not important for the main results whether CDS contracts are symmetric (both can terminate or neither can terminate) or asymmetric.
closer to the seller’s reservation value.\textsuperscript{21}

Analogously, a CDS seller and a CDS buyer Nash-bargain over the price such that the seller and the buyer get $\phi$ and $1 - \phi$ fractions of the total surplus, respectively. The buyer’s surplus is $V_{h[0,1]} - V_{h[0,0]}$ and the seller’s is $V_{h[0,1]} - V_{h[0,0]}$; thus, the CDS price is implicitly defined by
\begin{equation}
V_{h[0,1]} - V_{h[0,0]} = \phi \left( V_{h[0,1]} - V_{h[0,0]} + V_{h[0,1]} - V_{h[0,0]} \right).
\end{equation}

\subsection*{1.5 Value Functions}

To characterize the investors’ expected utilities, consider, for example, an $h[0,0]$ type. In a small time interval $[t, t+dt]$, he could (a) switch to an average-valuation (with probability $\gamma_adt$ and get utility 0), (b) become a bond owner (with probability $\lambda_b\mu_bdt$ and get $V_{h[1,0]} - p_b$), (c) become a CDS seller (with probability $\lambda_c\mu_cdt$ and get utility $V_{h[0,1]}$), or (d) remain an $h[0,0]$ type:
\begin{equation}
V_{h[0,0]} = (1 - rdt) \left( \gamma_adt(0) + \lambda_b\mu_bdt(V_{h[1,0]} - p_b) + \lambda_c\mu_cdtV_{h[0,1]} \\
+ (1 - \gamma_adt - \lambda_b\mu_bdt - \lambda_c\mu_cdt)V_{h[0,0]} \right).
\end{equation}

After simplifying and taking the continuous time limit, we get
\begin{equation}
rv_{h[0,0]} = \gamma_a(0 - V_{h[0,0]}) + \lambda_b\mu_b(V_{h[1,0]} - p_b - V_{h[0,0]}) + \lambda_c\mu_c(V_{h[0,1]} - V_{h[0,0]}).
\end{equation}

The value functions of the other agents are analogously derived in Appendix A.

\subsection*{1.6 Equilibrium}

\textbf{Definition 1.} A steady state equilibrium is population measures \{\mu_\tau\}_{\tau \in T}, prices \{p_b, p_c\}, entry decisions \{\rho\}, and value functions \{V_\tau\}_{\tau \in T} such that:

\begin{enumerate}
\item \{\mu_\tau\}_{\tau \in T} solve the steady state population flow equations in Table 3.
\item Market clearing conditions (3) and (4) hold.
\item Entry decisions, \{\rho\}, solve (1).
\item Bond and CDS prices, \{p_b, p_c\}, solve (5) and (6).
\item Agents’ value functions, \{V_\tau\}_{\tau \in T}, solve agents’ optimization problem given by (8), and (A.25)-(A.30).
\end{enumerate}

\textsuperscript{21}The buyer’s and the seller’s surpluses are $V_{h[1,0]} - V_{h[0,0]} - p_b$ and $p_b - V_{a[1,0]}$, respectively.
In a dynamic search model with multiple assets and more than two types of agents, as we can solve for the equilibrium only numerically, an equilibrium existence cannot be established for general parameter values (see Vayanos and Weill (2008) and Weill (2008)). However, Vayanos and Weill (2008) and Weill (2008) show that an existence of an equilibrium can be established when search frictions are small. Following their methodology, I show in the next proposition that a unique steady state equilibrium exists when search frictions are small (that is, \( \lambda_b \) and \( \lambda_c \) are large).

**Proposition 1.** Suppose

\[
x_b - \frac{(x_{ch} + (x_{cl} - 2y) \left( \frac{\lambda_{b\mu_b} + r + \gamma_d}{\lambda_{c\mu_a} + r + \gamma_u} \right))}{\left( r + \gamma_d + \gamma_u + \lambda_{b\mu_b} \phi_t + \lambda_{c\mu_a} \phi_h \right)} > 0.
\]

Then, for large \( \lambda_b \) and \( \lambda_c \), there exists a unique equilibrium.

The proof is given in Appendix A.

### 1.7 Liquidity Measures

I characterize three bond market liquidity variables. First, an illiquidity discount in the bond price is the difference between the bond price with and without search frictions (Propositions 2 and 3).

**Proposition 2.** If the bond market is frictionless (\( \lambda_b \rightarrow \infty \)), the bond price is given by

\[
p_b = \frac{\delta_b + x_b - y}{r} \quad (10)
\]

and the CDS market does not affect the bond market.

Proposition (2) shows that, without search frictions in the bond market, the bond price is the present value of high-valuation agent’s valuation of the bond. A bond owner—upon getting a liquidity shock—can sell instantly to another high-valuation trader. As a result, bonds are always in the hands of high-valuation agents and never held by agents who have a lower valuation. Since the bond price is a weighted average of the marginal valuations of different types of bond owners, and high-valuation investors are the only bond holders, the bond price is given by their valuation only. In this frictionless environment, the CDS market does not affect the bond market.

**Proposition 3.** The bond price is given by:

\[
p_b = \frac{\delta_b + x_b - y}{r} - \left[ \frac{\gamma_d}{r k} + \phi \left( \frac{x_b}{r k} \right) \right] - \left( \lambda_b \mu_{b} + r \right) \left( 1 - \phi \right) \lambda_c \mu_a \Delta_{h[0,1]},
\]

where \( \Delta_{h[0,1]} \) and \( k \) are defined by (A.62) and (A.39), respectively.
Proposition 3 shows that, with search frictions in the bond market, the bond price is lower than the frictionless price in (10). After receiving a liquidity shock, a bond owner does not value the bond as much and seeks to sell it. Because of search frictions, she is stuck with the bond for some time. When she does find a buyer, she accounts for the difficulty of locating another buyer and resorts to selling at a discounted price. Similarly, a potential bond buyer is only willing to buy at a low price as he anticipates the same trading friction when it is his turn to liquidate.

Thus, search costs create an illiquidity discount in the bond price given by the difference between (11) and the frictionless price (10), the sum of the second and third terms in (11). In particular, as bond buyers now have an outside option of providing liquidity in the CDS market (by selling CDS), the existence of CDS creates an additional discount in the bond price given by the third term.

**Definition 2.** The illiquidity discount, $d_b$, in the bond price is defined by the difference between the frictionless bond price (10) and the bond price with search frictions present in the bond market (11):

$$d_b \equiv \frac{\gamma_a}{r^k} x_b + \phi (\lambda_b \mu_{bs} + r) \frac{x_b}{r^k} + \frac{(\lambda_b \mu_{bb} + r)(1 - \phi)}{r^k} \lambda_c \mu_{cb} \Delta h_{[0,1]}.$$

I define two additional bond market liquidity variables. First, the bond bid-ask spread, denoted by $\omega_b$, is defined as the total trading surplus between a bond buyer and a seller: $\omega_b \equiv V_{h[1,0]} - V_{h[0,0]} + V_{a[1,0]}$. This definition captures the idea that, in an environment with dealers, a dealer intermediating the trade would charge bid-ask spreads proportional to the total gains from trade.\(^{22}\) Second, bond volume, denoted by $M_b$, is simply the total number of transactions, $M_b \equiv \lambda_b \mu_{h[0,0]} \mu_{a[1,0]}$.

### 2 Main Theoretical Results

#### 2.1 The Effect of Naked CDS on Bond Market Liquidity

The next proposition gives the main theoretical result of the paper. It shows that shorting bonds through naked CDS purchases increases bond market liquidity.

**Proposition 4** (The Spillover Effect). In the equilibrium of Proposition (1), the bond bid-ask spread ($\omega_b$) and the illiquidity discount ($d_b$) are smaller and the bond trading volume ($M_b$) is larger than in the environment without the CDS market.

\(^{22}\)If dealers, for example, capture the entire surplus, buyers are charged an ask-price equal to the buyers’ marginal valuation of the bond ($p_{ask} = V_{h[1,0]} - V_{h[0,0]}$) and sellers sell at a bid-price equal to their marginal valuation of the bond ($p_{bid} = V_{a[1,0]}$).
The proof is given in Appendix A, and the intuition is as follows. Allowing investors to short the underlying bonds through naked CDS purchases attracts investors who want to take the other side of the trade and hold long positions. These are the investors in the position of supplying liquidity into either market by buying bonds (from investors looking to liquidate) or selling CDS to short investors.\textsuperscript{23}

Because buying bonds and selling CDS are close substitutes for long investors and both involve search costs, long investors face an increasing returns to scale from searching simultaneously in both markets. The increasing returns to scale can arise from the fact that resources expended on pricing individual bonds help an investor price CDS relatively quickly and vice versa. Long investors, consequently, search and trade in both markets; as a result, the increase in the aggregate number of long investors translates to an increase in the number of bond buyers. Bond market liquidity, consequently, increases. This is the spillover effect; naked CDS buyers, by creating demand for liquidity, attract liquidity into credit markets that then spills over into the bond market.

\textbf{Model Implication on a Permanent CDS Ban}

A permanent CDS ban reverses the spillover effect.\textsuperscript{24} Liquidity suppliers are forced to exit the CDS market because their counterparties are banned from buying CDS. By exiting the CDS market, liquidity suppliers pull out from the bond market also. As a result, bond market liquidity and bond prices decrease. This prediction is my explanation for the observed decrease in bond market liquidity after the EU permanent CDS ban.

As discussed in Appendix F, the actual bans prevented CDS purchases for both speculating and hedging long positions correlated with the sovereign. In the model, consistent with the actual bans, both would be considered naked CDS purchases because the CDS buyer does not hold the underlying bonds.

\textbf{The Importance of CDS Search Frictions}

The first key ingredient for the spillover effect and, hence, in explaining the empirical patterns is search frictions in the CDS market. CDS has two opposing effects on bond market liquidity; which effect dominates depends on the extent of search frictions in the CDS market ($\lambda_c$). On the one hand, bond

\textsuperscript{23}Mechanically, high-valuation investors now have more profitable trading opportunities (in addition to buying bonds, they can now also sell CDS). The value of trading in credit markets as a whole increases. They enter at a greater rate until the marginal entrant is again indifferent. The result is an increase in the equilibrium entry rate and, consequently, the aggregate number of high-valuation investors.

\textsuperscript{24}A permanent CDS ban can be thought of setting the CDS matching efficiency ($\lambda_c$) to zero or, alternatively, as decreasing the flow of low-valuation investors ($F_l$) to zero because, except for bond owners, the ban made entering and buying CDS infinitely costly.
buyers are in a better bargaining position because they now have an outside option of providing liquidity in the CDS market (by selling CDS). This puts a downward pressure on the bond price and on bond market liquidity.\(^\text{25}\) On the other hand, because of the increase in the entry and number of high-valuation investors, bond sellers have a greater number of potential counterparties and, consequently, are also in a better bargaining position. This drives up bond market liquidity along with the bond price.

If the CDS market is frictionless \((\lambda_c \to \infty)\), these two opposite effects exactly offset one another. CDS induces a greater entry of long investors, but the additional entrants sell CDS immediately upon entry and, hence, do not end up simultaneously searching in the bond market.\(^\text{26}\) The aggregate number of high-valuation investors increases, but the increase does not translate to an increase in the number of bond buyers. Thus, in the absence of search frictions, CDS is redundant and has no effect on bond market liquidity. Proposition 5 formalizes this result.

**Proposition 5.** \(\lim_{\lambda_c \to \infty} d_b(\lambda_c) = d^\text{nocds}_b.\)

To illustrate the results of this section, Figure 2 plots bond market liquidity variables as functions of CDS market matching efficiency.\(^\text{27}\) Figure 3 illustrates the increase in the number of bond buyers and the decrease in the number sellers. Because the steady state measure of high-valuation investors is greater than both the bond supply and the steady state measure of low types, the direction of change in the number of bond buyers (not bond sellers) determines the direction of change in bond market trading volume. Consequently, the increase in the number of bond buyers results in an increase in trading volume.

**The Importance of Endogenous Entry**

The second key ingredient for the spillover effect is endogenous entry. If entry and, consequently, the aggregate number of high-valuation investors is fixed, the existence of naked CDS buyers instead decreases bond market liquidity. Some of the long investors who would have otherwise bought bonds migrate to the CDS market and sell CDS instead. For bond sellers, naked CDS buyers become a competition for the same set of investors who are in the position of providing liquidity in either market. Thus, bond sellers face

---

\(^{25}\) Recall, from \((11)\), the additional discount in the bond price due to CDS.

\(^{26}\) Mechanically, the increase in the equilibrium number of high-valuation investors, \((\rho^\text{ds} - \rho^\text{nocds}) \frac{\xi}{\rho}\), is exactly equal to the total demand for CDS (the measure of all low-valuation investors, including those who have purchased CDS: \(\frac{F_l}{\xi} = \mu_l(0,0) + \mu_l(0,-1)\)). Put differently, new high-valuation entrants replace one-to-one the bond buyers that migrate to the CDS market and sell CDS instead.

\(^{27}\) All the figures in Appendix D illustrate results for the range of parameter values of the x-axis variable for which Proposition 1 holds (except for the upper ranges of \(\lambda_b\) and \(\lambda_c\) since both can go to \(\infty\)).
a greater congestion externality and greater search costs, and the effect is a decrease in bond market liquidity. When entry is endogenous, however, the increase in the number of long investors not only replaces the bond buyers who migrate to the CDS market but also, due to search frictions in the CDS market, results in an even greater number of potential bond buyers.

**Short- versus Long-run Effects**

I refer to the results when entry is endogenous (the spillover effect) as a long-run effect and the results when entry is fixed as a short-run effect. The idea is that downscaling investment resources allocated to credit markets and scaling it back up is too costly (hence, fixed) in the short-run or if the change in one of the markets is only temporary. So any necessary reallocation occurs only locally within credit markets (i.e., at the intensive margin between assets that are close substitutes such as bonds and CDS). However, in the long-run or with a permanent change to one of the markets, investment resources get reallocated at a wider scope in and out of credit markets as a whole (i.e., at the extensive margin).

### 2.2 A Temporary Naked CDS Ban

This section models a temporary CDS ban and shows that the short-run effect explains the observed increase in bond market liquidity after Germany’s temporary ban.

I model a temporary naked CDS ban as a one-time unexpected drop in the number of naked CDS buyers (that is, a shock that perturbs the steady state). When the shock hits, the distribution of population measures switches to \{\bar{\mu}_t\}_{t\in T}, where the measure of naked CDS buyers is set zero (\bar{\mu}_{[0,0]} = 0), but the other elements of \{\bar{\mu}_t\}_{t\in T} are equal to their steady state values. To illustrate the short-run effect, I assume that the flow of high-valuation investors remains fixed as the economy rebounds back to the steady state equilibrium. Time is relabeled so that \( t = 0 \) corresponds to the time of the shock.

The time-varying equilibrium measure of \( h[0,0] \) agents from the shock back to the steady state is the solution to the following ordinary differential equation (ODE):

\[
\dot{\mu}_{h[0,0]}(t) = \rho F_h + \gamma u \mu_h(0,1)(t) - \left[ \gamma d \mu_{h[0,0]}(t) + (\lambda_h \mu_{bs}(t) + \lambda_c \mu_{cb}(t)) \mu_{h[0,0]}(t) \right].
\]

(13)

The initial condition is given by \{\mu_t(0)\}_{t\in T} = \{\bar{\mu}_t\}_{t\in T}, and the entry rate \( \rho \) is held fixed at the steady state level. The dynamics for the measures of other agents are analogously characterized in (A.85)–(A.91).
Agent $h[0,0]$’s value function evolves according to:

$$
\dot{V}_{h[0,0]}(t) = rV_{h[0,0]}(t) - [\gamma_d(0 - V_{h[0,0]}(t)) + \lambda_b\mu_{ba}(t) (V_{h[1,0]}(t) - V_{h[0,0]}(t) - p_b(t)) \\
+ \lambda_c\mu_{cb}(t)(V_{h[0,1]}(t) - V_{h[0,0]}(t))].
$$

(14)

The analogous ODEs for the other agents are in (A.92)–(A.98). Defining

$$
\Delta_{h[1,0]} \equiv V_{h[1,0]} - V_{h[0,0]} \quad \text{and} \quad \omega_c \equiv V_{h[0,1]} - V_{h[0,0]} + V_{h[0,-1]} - V_{h[0,0]},
$$

we can rewrite the value function ODEs in terms of $\Delta_{h[1,0]}$, $\omega_b$, and $\omega_c$. In turn, solutions for $\Delta_{h[1,0]}$, $\omega_b$, and $\omega_c$ are given in Proposition 6.

**Proposition 6.** Given the solution to the time-varying dynamics of agent measures, the dynamics for $\Delta_{h[1,0]}$ and $V_{a[1,0]}$ are given by:

$$
\Delta_{h[1,0]}(t) = \frac{\delta_b + x_b - y}{r} - \int_t^\infty e^{-r(s-t)} ((\gamma_d + \lambda_b\mu_{ba}(t)\phi) \omega_b(t) + \lambda_c\mu_{cb}(t)\phi\omega_c(t)) \, ds,
$$

(15)

$$
V_{a[1,0]}(t) = \frac{\delta_b - y}{r} + \int_t^\infty e^{-r(s-t)} \lambda_b\mu_{ba}(t) (1 - \phi) \omega_b(t) \, ds,
$$

(16)

where

$$
\begin{bmatrix}
\omega_b(t) \\
\omega_c(t)
\end{bmatrix} = \int_t^\infty e^{-\int_u^t A(u) \, du} \begin{bmatrix}
x_b \\
x_c + x_{ch} - 2y
\end{bmatrix} \, ds,
$$

(17)

and $A(t)$ is defined by (A.105).

**Results**

Figures 4 and 5 plot the short-run dynamics of population measures and bond market liquidity variables from the CDS ban at $t = 0$ back to the steady state. The sudden drop in the number of naked CDS buyers frees up long investors who would have otherwise sold CDS to them. Long investors, as a result, temporarily substitute providing liquidity in the CDS market with providing liquidity in the bond market. In turn, bond sellers temporarily benefit from the ban as they now locate buyers quickly and face lower search costs.\(^{28}\)

Thus, a temporary CDS ban reverses the short-run effect described earlier. Long investors do not exit entirely from credit markets (as was the case with a permanent ban); instead, they resort to temporarily trading in the bond market. The immediate effect is an increase in bond market liquidity. This prediction is consistent with the observed increase in bond market liquidity following the German temporary ban.

As described in detail in Appendix F, the temporary ban was implemented by the German government and applicable to German institutions. Although,\(^{28}\)

\(^{28}\)As the ban is lifted, the number of traders searching to buy CDS increases until the fraction of CDS buyers who finds a CDS seller equals the flow of new low-valuation agents entering the economy.
at first glance, the ban may seem limited in scope, it applies to one of the major players in a market already concentrated among few dealers: Deutsche Bank. The ban, consequently, would have affected a nontrivial subset of the CDS market. A more realistic specification is to set the number of naked CDS buyers—instead of to zero—to $\bar{\mu}_{[0,0]} = (1 - x)\mu_{[0,0]}$, where $x$ is the fraction of CDS demand originating from Germany. Results, however, would not change qualitatively. Also, the temporary ban lasted for about two months, whereas it lasts only an instant in the model. Again, the direction of the results would be the same if it were to last longer. The focus of this section is instead to qualitatively capture the immediate reaction to a ban when the entry margin has not had enough time to adjust.

An Implicit Cost of Entry

The short-run effect arises from holding the entry rate fixed. This assumption captures, in a reduced form, an adjustment cost of entry. Although I do not explicitly incorporate an adjustment cost of entry, equation (18) illustrates one possible way of incorporating it. Now, in addition to comparing the value of entering $V_{h[0,0]}(\rho)$ with the outside investment opportunity $O_h$, high-valuation agents account for a cost of entry, $c(\rho)$, that varies with the entry rate:

$$\rho = \begin{cases} 1 & V_{h[0,0]}(\rho) - c(\rho) > O_h \\ [0, 1] & V_{h[0,0]}(\rho) - c(\rho) = O_h \\ 0 & V_{h[0,0]}(\rho) - c(\rho) < O_h, \end{cases}$$

(18)

where $c'(\rho) \geq 0$, $c''(\rho) > 0$, $c(0) \geq 0$, and $c(1) < \infty$. Figure 6 illustrates an example of such a cost function. We can also back out the cost of entry short-run dynamics from the dynamics of $V_{h[0,0]}(\rho^{**})$ (see Figure 6).

3 Additional Results

Section 3.1 numerically analyzes how exogenously changing market liquidity and funding liquidity—through their effect on endogenous liquidity of both markets—affect prices and the CDS-bond basis. Then, Section 3.2 extends the model to allow for bond holders to buy CDS and hold covered CDS positions. Section 3.3 calibrates the model and numerically illustrates the marginal effects of covered and naked CDS positions.

3.1 Additional Empirical Predictions

First, I characterize closed-form solutions for the CDS price and CDS market liquidity.
Proposition 7. The price of a CDS contract is given by:

\[ p_c = \delta_c + x_{cl} - y - \frac{1 - \phi}{\phi} (r + \gamma_u + \lambda_c \mu_{cs}) \Delta_{h[0,1]} \]  

(19)

Proposition 8. The CDS price in a frictionless environment \((\lambda_c, \lambda_b \to \infty)\) is given by:

\[ \lim_{\lambda_c, \lambda_b \to \infty} p_c = \delta_c - x_{ch} + y \]  

(20)

**Intuition.** High-valuation investors are the marginal sellers of CDS contracts. As a result, in a frictionless environment, the CDS price is given by the high-valuation investors’ flow cost of providing insurance.

It is straight-forward to show that CDS contracts are more expensive with search frictions than without: \(p_c > \lim_{\lambda_c, \lambda_b \to \infty} p_c\). As with bond illiquidity discount, we define CDS illiquidity as the wedge between the CDS price with and without search frictions. In the case of CDS, it is an illiquidity premium, not a discount.

**Definition 3.** The illiquidity premium, \(d_c\), in the CDS price is defined as the difference between the frictionless CDS price and the price with search frictions:

\[ d_c \equiv p_c - \lim_{\lambda_c, \lambda_b \to \infty} p_c = x_{ch} + x_{cl} - 2y - \frac{1 - \phi}{\phi} (r + \gamma_u + \lambda_c \mu_{cs}) \Delta_{h[0,1]} \]  

(21)

The CDS bid-ask spread is defined, analogous to the bond bid-ask spread, as the total trading surplus: \(\omega_c \equiv V_{h[0,1]} - V_{h[0,0]} + V_{(0,-1]} - V_{(0,0)}\). This definition captures the idea that, in an environment with dealers, dealers would charge bid-ask spreads proportional to the total trading surplus.

Results 1–3 analyze how bond and CDS market liquidity and prices change with respect to the CDS market matching efficiency \((\lambda_c)\), bond market matching efficiency \((\lambda_b)\), and value of the outside option, \(O_h\). They are derived numerically using the parameter values in Table 6.

**Result 1.** The effect of an exogenous increase in CDS market liquidity \((\lambda_c)\).

1. Bond market liquidity deteriorates: The illiquidity discount \((d_b)\) and the bid-ask spread \((\omega_b)\) increase while the trading volume \((M_b)\) decreases.
2. CDS market liquidity increases. The illiquidity premium \((d_c)\) and the bid-ask spread \((\omega_c)\) decrease while the trading volume \((M_c)\) increases.
3. The bond price \((p_b)\) decreases.
4. The CDS price \((p_c)\) decreases.
Result 1 can be seen from Figures 2 and 7. An increase in the CDS meeting intensity ($\lambda_c$) increases CDS market liquidity, which is intuitive, but it also decreases bond market liquidity. This is because the CDS market had a positive externality on bond market liquidity in the presence of search frictions in the CDS market. Lowering CDS search frictions reverses the positive externality.

Because of the opposite changes in bond and CDS market liquidity, the CDS price and the bond yield also change in opposite directions: CDS becomes cheaper (which in the data may be perceived as a decrease in credit risk), but the bond yield increases. Lastly, it is generally not obvious whether CDS market liquidity increases or decreases the CDS price and, hence, whether the CDS buyer or seller that extracts the rent. The model shows that the CDS seller captures the spread.

Result 2, illustrated in Figure 8, shows that the effects of an exogenous increase in bond market liquidity are exactly the opposite of an exogenous increase in CDS market liquidity.

**Result 2.** The effect of an exogenous increase in bond market liquidity ($\lambda_b$).

1. Bond market liquidity increases. The illiquidity discount ($d_b$) and the bid-ask spread ($\omega_b$) decrease while the trading volume ($M_b$) increases.
2. CDS market liquidity decreases. The illiquidity premium ($d_c$) and the bid-ask spread ($\omega_c$) increase while the trading volume ($M_c$) decreases.
3. The bond price ($p_b$) increases.
4. The CDS price ($p_c$) increases.

Result 3, illustrated in Figure 9, covers the effect of exogenously increasing funding liquidity. In particular, it relaxes the entry constraint by decreasing the value of the outside option, $O_h$.

**Result 3.** The effect of an exogenous increase in funding liquidity (a decrease in $O_h$).

1. Bond market liquidity increases. The illiquidity discount ($d_b$) and the bid-ask spread ($\omega_b$) decrease while trading volume ($M_b$) increases.
2. CDS market liquidity generally increases. The illiquidity premium ($d_c$) decreases while the trading volume ($M_c$) increases. The change in the bid-ask spread ($\omega_c$), however, is not monotonic.
3. The bond price ($p_b$) increases.
4. The CDS price ($p_c$) decreases.
Implications for the CDS-Bond Basis

The CDS-bond basis is the CDS spread (the price of a CDS contract) minus the bond yield, adjusting for a risk-free rate. It captures the pricing difference of the same underlying credit risk by the bond and CDS markets. In a frictionless world, the difference and, consequently, the basis should be zero. A large body of empirical papers document a persistent deviation of the basis from zero. Consequently, the recent CDS literature has focused on the determinants of the CDS-bond basis and how the relative liquidity of bond and CDS markets affects it. Empirical analysis, however, is limited by endogeneity problems because asset prices and liquidity are interdependent, within as well as across markets. Using Results 1–3, Corollary 1 disentangles how exogenously changing market liquidity and funding liquidity affect the CDS-bond basis (see Figure 10).

**Corollary 1.** The effects of exogenous variations in market liquidity and funding liquidity on the CDS-bond basis.

1. An exogenous increase in CDS market liquidity decreases the CDS-bond basis (the basis becomes more negative).

2. An exogenous increase in bond market liquidity increases the CDS-bond basis (the basis becomes more positive).

3. Exogenously relaxing the entry constraint increases the CDS-bond basis (the basis becomes more positive).

From Result 1, an exogenous increase in CDS market liquidity (through its effect on liquidity of both markets) decreases the CDS spread but increases the bond yield. As a result, the basis declines (Corollary 1.1). The effect of an exogenous increase in bond market liquidity is exactly the opposite: The basis increases (Corollary 1.2). This is consistent with Bai and Collin-Dufresne (2011) who find that bond market liquidity increases the basis.\(^{29}\) However, they do not control for CDS market liquidity. Arce, Mayordomo, and Peña (2012) find that bond market liquidity still increases the basis, controlling for CDS market liquidity.

Finally, the model predicts that exogenously increasing funding liquidity increases the basis (Corollary 1.3). From Result 3, the CDS spread decreases, but the bond yield decreases also. Thus, the effect on the basis can be ambiguous. However, for the parameter values used (which I argue are reasonable in Section 3.3), the model predicts an increase in the basis.

\(^{29}\)They find bond illiquidity measured by bid-ask spreads decreases the basis.
3.2 Extension: Covered CDS

This section extends the model to allow for bondholders to purchase CDS protection and, consequently, hold covered CDS positions $a[1,-1]$. The limitation of the benchmark environment with three types of investor valuations is that covered CDS positions do not naturally arise. The only bondholders who are candidates to buy CDS (hence, hold covered CDS positions) are the average-valuation bondholders. But given Assumption 1 that $2y > x_{ch}$, average-valuation investors do not profit from buying CDS (whether they hold bonds or not); only low-valuation investors benefit from buying CDS. Thus, to “force” average-valuation bondholders to buy CDS (while they are looking to sell their bonds), I assume that a flow benefit $y_{1,-1}$ exists just for the covered CDS position. Such benefit can arise from, for example, relaxed capital requirements from hedging bond positions.

I describe next how allowing for covered CDS positions changes the environment and trading strategies for each investor type. All described trading opportunities (and the corresponding intensities and surpluses) are reflected in the value and measure functions in Appendix B.

Covered CDS buyers ($a[1,-1]$ type)

An average-valuation bondholders can now, in addition to searching for a bond buyer, simultaneously search for a CDS seller. If she bumps into a CDS seller before selling her bond, she buys CDS at price $p_{c,1,-1}$ and becomes a covered CDS holder. This matching opportunity is reflected in the value and measure functions of the bond buyer: (B.6) and Table 4. She remains a covered CDS holder until she sells her bond. Upon selling her bond, she terminates her CDS contract and exits the economy.

CDS sellers ($h[0,0]$ type)

A CDS seller can bump into either a naked CDS buyer (low-valuation nonowner) or a covered CDS buyer (average-valuation bond owner). Trading gains and, hence, bargained CDS prices now depend on the type of the CDS buyer (covered versus naked). Thus, I categorize CDS sellers by their counter-parties. A CDS seller (i.e., with position $[\theta_{b}, \theta_{c}] = [0, 1]$) whose counterparty is a naked CDS buyer is denoted by $\tau = i[0, 1]^{0,-1}$ for $i \in \{h, a\}$. A CDS seller whose counterparty is a covered CDS buyer is denoted by $\tau = i[0, 1]^{1,-1}$ for $i \in \{h, a\}$.

As before, the CDS price, $p_{c}$, bargained between a CDS seller and a naked CDS buyer is determined implicitly by

$$V_{h[0,1]^{0,-1}} - V_{h[0,0]} = \phi \left( V_{i[0,-1]} - V_{i[0,0]} + V_{h[0,1]^{0,-1}} - V_{h[0,0]} \right).$$ (22)
The CDS price bargained between a CDS seller and a covered CDS buyer, \( p_{c[1,1]} \), is instead determined by
\[
V_{h[0,1][1,-1]} - V_{h[0,0]} = \phi \left( V_{a[1,-1]} - V_{a[1,0]} + V_{h[0,1][1,-1]} - V_{h[0,0]} \right). \tag{23}
\]

As before, a CDS seller upon switching to an average-valuation seeks to unwind her long position by searching for another CDS seller to take over her position. Thus, among \( h[0,0] \) investors, who are searching to sell CDS, some \textit{directly} bump into a naked CDS buyer. Others instead \textit{indirectly} sell to a naked CDS buyer by finding and taking over from an average-valuation investor her previously established long position with a naked CDS buyer. In this case, the average-valuation agent effectively intermediates the transaction between high- and low-valuation agents. For the \( h[0,0] \) investor, this indirect match with a naked CDS buyer occurs with probability \( \lambda_c \mu_{a[0,1][0,-1]} \). Since he gets the same price in both the direct and indirect matches, with total intensity \( \lambda_c (\mu_{[0,0]} + \mu_{a[0,1][0,-1]}) \), his trading surplus is \( V_{h[0,1][0,-1]} - V_{h[0,0]} \).

Analogously, the \( h[0,0] \) investor will meet a covered CDS buyer with intensity \( \lambda_c (\mu_{a[1,0]} + \mu_{a[0,1][1,-1]}) \) either directly or indirectly by taking over an average-valuation investor’s long position with a covered CDS buyer. In both cases, the trading surplus is \( V_{h[0,1][1,-1]} - V_{h[0,0]} \).

Bond buyers (\( h[0,0] \) type)

As in the benchmark environment, in addition to selling CDS, \( h[0,0] \) investors enter the bond market to buy bonds. Now, a bond buyer’s trading surplus and the price he pays for the bond depend on the CDS position of the bond seller. With intensity \( \lambda_b \mu_{a[1,0]} \), he meets a bond seller who has not purchased CDS, in which case the bargained bond price and the trading surplus are the same as before. If he meets a bond seller who has purchased CDS (which occurs with intensity \( \lambda_b \mu_{a[1,-1]} \)), the bargained bond price is instead given by
\[
p_{b[1,-1]} = \phi V_{a[1,-1]} + (1 - \phi) (V_{h[1,0]} - V_{h[0,0]}). \tag{24}
\]
The trading surpluses to the buyer and the seller are \( V_{b[1,0]} - V_{b[0,0]} - p_{b[1,-1]} \) and \( 0 - V_{a[1,-1]} - p_{b[1,-1]} \), respectively.

CDS buyers (\( l[0,-1] \) and \( a[1,-1] \) types)

As before, a naked CDS buyer upon switching to an average-valuation cancels her contract and exits the economy. This forces her counterparty to search all over again if he was a high-valuation investor. If her counterparty is instead an average-valuation agent seeking to get out of the contract, her counterparty simply exits the economy. In contrast to naked CDS buyers, the only
reason a covered CDS buyer terminates her contract is if she sells her bond.\textsuperscript{30} The termination affects her counterparty analogously to a termination by a naked CDS buyer.

Finally, the market clearing conditions for the bond and CDS markets, (B.1) and (B.2), reflect the changes in the environment due to the addition of covered CDS positions.

Results

I first describe the marginal effect of covered CDS. The results are derived numerically using the parameter values in Table 6 and are shown in Figure 11. First, due to the ability to hedge bonds with CDS, investors are more willing to buy and hold bonds. The result is an increase in the value of the bond and, consequently, a decrease in the illiquidity discount. Second, as both the bond seller and buyer have outside options of trading in the CDS market, the total surplus to trading a bond is lower. This means narrower bond bid-ask spreads. Third, as some transactions that would have taken place in the bond market now take place in the CDS market, bond volume is lower.

Recall, in the environment with naked CDS only, CDS affects the bond market only in the presence of CDS search frictions and is otherwise redundant. In contrast, the above described effects of covered CDS hold irrespective of the extent of search frictions in the CDS market. In particular, bond market liquidity variables all approach a new steady state level as the CDS market becomes frictionless ($\lambda_c \to \infty$). These effects can be seen in Figure 11 by comparing the environment with covered CDS (in thin dashed) with the environment without CDS (in thick dashed).

The marginal effects of allowing covered CDS positions also hold both in the short-run (i.e., when entry is fixed) and in the long-run. Figure 12 shows the marginal effect of allowing covered CDS positions when entry is fixed. Thus, the effect of covered CDS on the bond market cannot on its own explain why different bans had different effects on bond market liquidity. The reason that the short- and long-run effects of naked CDS are different, but those of covered CDS are the same is as follows. With the addition of naked CDS buyers, an entirely fresh set of short investors are introduced into the economy. Hence, whether entry of their would-be natural counterparties is endogenous or not matters. Allowing covered CDS positions, in contrast, does not introduce into the economy new investors but only new positions for investors that had already existed in the economy (i.e., bond sellers).

Comparing the setting with both covered and naked positions with the setting with just covered CDS isolates the additional effect of naked CDS

\textsuperscript{30}Recall that average-valuation investors who already entered the economy do not switch to a high or a low type.
positions relative to a benchmark with covered CDS. Figure 11 shows, in thin dashed lines, the results with just covered CDS and, in thin solid lines, the results with both. By contrasting the two, the marginal effect of naked CDS (relative to a benchmark with covered CDS) is exactly the same as in Section 2: The bond price is higher and closer to the fundamental, bond bid-ask spreads are narrower, and bond volume is greater. Thus, the results outlined in Section 2 still hold in the presence of covered CDS positions.

3.3 Calibration

This section calibrates the model environment with both covered and naked CDS trading and numerically illustrates the marginal effects of naked CDS (Section 2) and covered CDS positions (Section 3.2). Parameters are calibrated to match data moments reported in Sambalaibat (2014) for an average sovereign over the period 2008–2012.

Table 6 shows the calibrated parameter values. The risk-free rate is set at 4%. Bond supply, $S$, is normalized to 1. Sellers and buyers have an equal bargaining power of $\phi = \frac{1}{2}$. The bond coupon flow, $\delta_b$, and the flow benefit of buying CDS, $\delta_c$, are both normalized to 1%.

The flow of high-valuation investors, $F_h$, is calibrated to CDS outstanding as a percent of debt outstanding of around 30%. The flow of low-valuation investors, $F_l$, is set to satisfy $\frac{\alpha F_h}{\gamma_d} \geq s + \frac{F_l}{\gamma_u}$. How large this difference is (i.e., the inequality) affects the difference between the time it takes to buy and the time it takes to sell in bond and CDS markets. This difference, in contrast, has a smaller effect on bond and CDS volume. Switching intensities, $\gamma_d$ and $\gamma_u$, are calibrated to match annual bond volume to debt outstanding—which is only available for Italy and is 50%—and annual CDS volume as a percent of CDS outstanding of around 80% for an average sovereign.

Matching efficiencies, $\lambda_b$ and $\lambda_c$, are calibrated to search times in bond and CDS markets (the expected days to find a buyer or a seller). What these search times should be is not obvious and is not well documented. Vayanos and Weill (2008) argue it should between a couple hours and a few days for U.S. government bonds. Since the U.S. government bond market is the most liquid bond market, its search times serve as a lower bound for an average sovereign. From Sambalaibat (2014), the proportional bond bid-ask spread is about 1% for an average sovereign and 0.03% for U.S. government bonds. Thus, search times for an average government bond market may be as much as 30 times longer than for the U.S. Treasury market. On the other hand, the expected number of days to trade for a seller (buyer) can be computed as the ratio of the number of sellers (buyers) to the number of trades per day. Siriwardane (2015) reports about 1,700 counterparties in the CDS market. Sambalaibat (2014) reports an average of 12.37 transactions per day for sovereign reference names. Using 1,700 as the number of potential
counterparties results in 170 days to find a buyer or a seller. This provides an upper bound because 10–15 large dealers account for a majority of CDS transactions. I calibrate to the lower end of the possible range of few days to 170 days: 30–50 days. Siriwardane (2015) also reports that the sell side of the CDS market is more concentrated than the buy side; thus, selling CDS should be quicker than buying CDS. Table 7 shows the resulting calibration moments.

Hedging benefits \( x_b, x_{ch}, \) and \( x_{cl} \) and the cost of risk bearing \( (y \) and \( y_{1,-1} ) \) are set so that the expected bond return is about 6%, the bond bid-ask spread (as a percent of the bond price) is 2%, the CDS bid-ask spread (as a percent of the CDS price) is 13%, the CDS price is 2%, and so that the trading gain for a bond owner from buying CDS is positive \( (\omega c[1,0] > 0) \).

Results

Tables 7 and 8 show the calibration results and the counterfactuals of what bond and CDS market liquidity and prices would be if there were no CDS market at all (column 3), only naked CDS transactions (column 4), and only covered CDS transactions (column 5).\(^{31}\)

I highlight three results. First, the effects of CDS on bond market liquidity show up more through prices (that is, bid-ask spreads and illiquidity premia) than through trading volume. Second, the calibration of the model with reasonable parameter values generates changes in the bond bid-ask spread that are economically significant and consistent with the changes around the CDS bans documented in Sambalaibat (2014). The bond bid-ask spread (as a percent of the bond price) and the bond expected return are 27% and 5% higher in the no-CDS environment than in the environment with both covered and naked CDS trading. This implies that a permanent CDS ban widens the bond bid-ask spread by up to 27%. The temporary ban exercise (Figure 5) generated with the above calibration values shows that the bond bid-ask spread narrows by about 6.5%.

Third, comparing the relative importance of covered and naked CDS positions, most of the decrease in the bond bid-ask spread and the expected return comes from the effect of naked CDS, not covered CDS positions. This is not surprising given that, as a result of the calibration, the majority of CDS outstanding (99.2%) are naked purchases.

\(^{31}\)In “Both” column, the environment with both covered and naked CDS positions, bond-sellers hedged with CDS sell their bond at a price different from those who are not hedged. I show the weighted average of these two prices, weighted by the volume of each transaction. Similarly, the CDS price shown is a weighted average of the prices paid by naked and covered CDS buyers. Bid-ask spreads are treated similarly.
4 Discussion

In this section, I discuss some of the assumptions of the paper.

CDS contracts exist in the model because investors cannot directly short bonds, and buying CDS is a way to short credit risk. As discussed in the introduction, this assumption is motivated by the derivative and fixed supply features of CDS and bonds and the fact that CDS is a standardized instrument. In addition, in contrast to shorting the underlying asset through CDS, shorting a bond (as well as unwinding the bond position) involves multiple search processes. An investor has to first search for a counterparty in the repo market to borrow a bond from. Then, she goes to the cash bond market to search for a counterparty to sell the bond to. CDS instead involves just one search process. Unwinding the short bond position requires searching for the specific bond that was borrowed in the first place. Whereas, with CDS, an investor can simply terminate the contract. Allowing bond shorting would simply change the benchmark environment, and whether the benchmark environment includes bond shorting or not, the marginal effect of naked CDS buyers will be the same.

In the model, as in standard search models of financial assets, investors can hold only zero or one unit of the bond and the CDS. Allowing investors to hold multiple units of bond is less important because it is equivalent to normalizing the size of a bond trade. Allowing multiple units of CDS, however, would get at the derivative versus fixed supply features of CDS and bonds and might endogenously give rise to heterogeneous trading costs of shorting through the bond and CDS markets. Nevertheless, allowing multiple CDS units would most likely make the spillover effect even stronger.

I assumed that long investors can simultaneously search in both the bond and the CDS market and, as a result, face increasing returns to scale from doing so. This captures the idea that resources expended on pricing individual bonds help an investor price CDS relatively quickly and vice versa. In the model, they transact based on wherever they find a counterparty first and do not have to choose to search in one market over the other or be indifferent between the two markets. Imposing that investors choose one of the markets typically gives rise to market segmentation (see Vayanos and Wang (2007)). Imposing such constraint is somewhat unrealistic given that we do not observe market segmentation and instead see traders simultaneously participating in both markets.

Imposing investors choose one of the markets is analogous to endogenizing search intensities. Thus, for similar reasons, endogenizing search intensities—that is, matching efficiencies ($\lambda_b$, $\lambda_c$)—results in market segmentation and is, hence, unsatisfactory.$^{32}$

$^{32}$One way to ensure that both markets can coexist is to impose additional exogenous
I have assumed that all investors can participate in both markets. In reality, some market participants, such as retail investors, can trade in the bond market but not in the CDS market. To that extent, I examine the effect of CDS only on the subset of bond traders who can also trade in the CDS market. However, if the effects get passed down to the rest of the bond market (i.e., those who cannot trade CDS), then CDS would indirectly affect them also and in the ways described in the paper. As for CDS traders, most if not all CDS participants can also trade in the bond market. Thus, I capture the entire CDS market.

Dealers are absent in the model. Dealers, in reality, do not eliminate search costs. They instead charge bid-ask spreads proportional to the search costs customers would incur if they were to bypass dealers and search for each other directly. Thus, bid-ask spreads in my model capture the intermediation spread that dealers would charge had they been explicitly incorporated in the model.

Informational frictions do not play a role in the model. In reality, CDS might also affect the bond market through an informational channel. For example, CDS might reduce bond market liquidity due to the information asymmetry between investors and the bond issuer (a sovereign, for example). As an instrument to trade on negative news, shorting credit risk through the CDS market may amplify a “run” on sovereign bond markets which then leads to a further liquidity dry-up in the bond market. Although plausible, this mechanism cannot on its own explain why different CDS bans have different effects on bond market liquidity.

Illiquidity can also arise from asymmetric information amongst traders, as in Kyle (1985) and Glosten and Milgrom (1985). The search framework is better suited for OTC markets and, in particular, sovereign bond markets for two reasons. First, illiquidity in bond markets is more due to fragmentation of trades across heterogeneous bonds. Second, asymmetric information and insider trading is less severe with respect to governments than with respect to individual firms.

Finally, the model has no aggregate shocks. Although beyond the focus of the paper, it would be interesting to analyze which market responds faster and more strongly to an aggregate shock. Such analysis would shed light on the relative price discovery and how the relative liquidity of the two markets affects it.

Heterogeneity on traders and markets. As a result, a clientele effect can arise where one type of investors trade in one market and the other types trade in the other. This is an additional complication and not necessary in my model. Nevertheless, as long as entry is endogenous, in the equilibrium where both markets exist (which is what we see in reality), naked CDS buyers should still increase the number of long investors.
5 Conclusion

I build a search model of OTC bond and CDS markets featuring an endogenous liquidity interaction between the two markets and endogenous funding liquidity. I show that, in the long-run, trading activity and liquidity in the CDS market spill over into the bond market and increase bond market liquidity. This spillover effect arises from search frictions and from endogenizing the entry and aggregate number of investors, which, in standard search models, is held fixed. In the short-run, however, the entry rate is sticky and is unaffected by the CDS market. Consequently, introducing the CDS market decreases liquidity in the bond market.

These effects are economically relevant. Sambalaibat (2014) documents that different naked CDS bans implemented in Europe (one permanent and the other temporary) had exactly opposite effects on bond market liquidity. The long-run and short-run effects help explain the observed changes in bond market liquidity around these CDS bans.
A Proofs

To simplify notation, I define \( q_{ca} \equiv \lambda_c \mu_h(0,0) \), \( q_{cb} \equiv \lambda_c \mu_{l}(0,0) \), \( q_{bs} \equiv \lambda_b \mu_{a}(1,0) \), \( q_{bb} \equiv \lambda_b \mu_h(0,0) \), \( \phi_h \equiv \phi \), and \( \phi_l \equiv 1 - \phi \).

Table 3: Flow-ins and outs

<table>
<thead>
<tr>
<th>Type</th>
<th>Flow-in = Flow-out</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h[0,0] )</td>
<td>( \rho F_h + \gamma a \mu_h[0,1] = \gamma d \mu_h[0,0] + (\lambda b \mu_{bs} + \lambda_c \mu_{cb}) \mu_h[0,0] )</td>
</tr>
<tr>
<td>( l[0,0] )</td>
<td>( F_l = \gamma a \mu_l[0,0] + \lambda_c \mu_{ca} \mu_l[0,0] )</td>
</tr>
<tr>
<td>( h[1,0] )</td>
<td>( \lambda_b \mu_h \mu_h[0,0] = \gamma d \mu h[1,0] )</td>
</tr>
<tr>
<td>( a[1,0] )</td>
<td>( \gamma d \mu h[1,0] = \lambda b \mu_h \mu_a[1,0] )</td>
</tr>
<tr>
<td>( h[0,1] )</td>
<td>( \lambda c \mu_{cb} \mu_h[0,0] = \gamma d \mu h[0,1] + \gamma a \mu h[0,1] )</td>
</tr>
<tr>
<td>( a[0,1] )</td>
<td>( \gamma d \mu h[0,1] = \gamma a \mu_a[0,1] + \lambda c \mu_{ca} \mu_a[0,1] )</td>
</tr>
<tr>
<td>( l[0,-1] )</td>
<td>( \lambda c \mu_{sa} \mu[0,0] = \gamma a \mu[0,-1] )</td>
</tr>
</tbody>
</table>

Agents’ flow-value equations are analogously derived to (8):

\[
\begin{align*}
  rV_{l[0,0]} &= \gamma a (0 - V_{l[0,0]}) + q_{ca} (V_{l[0,-1]} - V_{l[0,0]}) \quad (A.25) \\
  rV_{h[1,0]} &= \delta b + x_b - y + \gamma d (V_{a[1,0]} - V_{h[1,0]}) \quad (A.26) \\
  rV_{a[0,1]} &= \delta b - y + q_{bs} (0 - V_{a[1,0]} + p_b) \quad (A.27) \\
  rV_{h[0,1]} &= p_c - (\delta c - x_{ch}) - y + \gamma d (V_{a[0,1]} - V_{h[0,1]}) + \gamma a (V_{h[0,0]} - V_{h[0,1]}) \quad (A.28) \\
  rV_{a[0,1]} &= p_c - \delta c - y + q_{cs} (0 - V_{a[0,1]} + p_a) + \gamma a (0 - V_{a[0,1]}) \quad (A.29) \\
  rV_{l[0,-1]} &= -p_c + (\delta c + x_{ca}) - y + \gamma a (0 - V_{l[0,-1]}) \quad (A.30)
\end{align*}
\]

**Proof of Proposition 1.** The proof of uniqueness is shown in Lemma 1, and the proof of existence is shown in Lemma 2. \( \square \)

**Lemma 1.** Suppose (9) holds, then the steady state equilibrium is unique.

**Proof.** First fix \( \rho \). Then, using the inflow outflow equations and the market clearing conditions, the population measures (except for \( \mu_h[0,0] \)) can be solved as functions of \( \mu_h[0,0] \):

\[
\begin{align*}
  \mu_l[0,0] &= \frac{F_l}{\gamma a + \lambda c \mu_h[0,0]} \quad (A.31) \\
  \mu_h[1,0] &= \frac{S \lambda b \mu_h[0,0]}{\lambda b \mu_h[0,0] + \gamma d} \quad (A.32) \\
  \mu_a[1,0] &= S - \frac{S \lambda b \mu_h[0,0]}{\lambda b \mu_h[0,0] + \gamma d} \quad (A.33) \\
  \mu_h[0,1] &= \frac{\gamma a (\lambda c \mu_h[0,0] + \gamma d + \gamma a)}{\gamma d F_l \lambda c \mu_h[0,0]} \quad (A.34) \\
  \mu_a[0,1] &= \frac{\gamma a (\lambda c \mu_h[0,0] + \gamma a) (\lambda c \mu_h[0,0] + \gamma d + \gamma a)}{\gamma a (\lambda c \mu_h[0,0] + \gamma a)} \quad (A.35) \\
  \mu_l[0,-1] &= \frac{\lambda c F_l \mu_h[0,0]}{\gamma a (\lambda c \mu_h[0,0] + \gamma a)} \quad (A.36)
\end{align*}
\]
\( \mu_{h[0,0]} \) itself is a solution to:

\[
\rho F_h - \gamma_d \mu_{h[0,0]} \left( \frac{S \lambda_b}{\lambda_b \mu_{h[0,0]} + \gamma_d} + \frac{\lambda_c F_l}{\gamma_u (\lambda_c \mu_{h[0,0]} + \gamma_d + \gamma_u) + 1} \right) = 0. \tag{A.37}
\]

The LHS of (A.37) is positive at \( \mu_{h[0,0]} = 0 \), decreasing in \( \mu_{h[0,0]} \), and negative for large \( \mu_{h[0,0]} \). Thus, (A.37) uniquely determines \( \mu_{h[0,0]} \) and has a positive solution, while the other \( \mu \)'s are uniquely determined by (A.31)–(A.35). Next, once the \( \mu \)'s are solved, the value functions and prices are uniquely determined by a linear system of equations: (8), (A.25)–(A.30), and (5)–(6).

We are left with the endogenous entry decision (1). There are three cases: two corner solutions, \( \rho = 0 \) and \( \rho = 1 \), and an interior solution. Next, I show that \( V_{h[0,0]} \) is strictly decreasing in \( \rho \), which will imply that, under each case, the equilibrium is unique. The derivation in the proof of existence shows that

\[
V_{h[0,0]} = \frac{q_{bs} x_b \phi + \Delta_{h[0,1]} q_{cb} (r + \gamma_d + q_{lb}(1 - \phi))}{(r + \gamma_d) k}, \tag{A.38}
\]

where

\[
k \equiv r + \gamma_d + \lambda_b \mu_{bs} \phi + \lambda_b \mu_{lb}(1 - \phi), \tag{A.39}
\]

and \( \Delta_{h[0,1]} \) is given by (A.62). None of the population measures, other than \( \mu_{h[0,0]} \), directly depend on \( \rho \), only indirectly through \( \mu_{h[0,0]} \). Thus, consider the derivative of \( V_{h[0,0]} \) with respect to \( \rho \):

\[
\frac{\partial V_{h[0,0]}(\rho)}{\partial \rho} = \frac{\partial V_{h[0,0]}(\rho)}{\partial \mu_{bs}} \frac{\partial \mu_{bs}}{\partial \mu_{h[0,0]}} + \frac{\partial V_{h[0,0]}(\rho)}{\partial \mu_{lb}} \frac{\partial \mu_{lb}}{\partial \mu_{h[0,0]}} + \frac{\partial V_{h[0,0]}(\rho)}{\partial \mu_{h[0,0]}} + \frac{\partial V_{h[0,0]}(\rho)}{\partial \mu_{cs}} \frac{\partial \mu_{cs}}{\partial \mu_{h[0,0]}}.
\tag{A.40}
\]

Next, I derive \( \frac{\partial V_{h[0,0]}}{\partial \mu_{bs}} \), \( \frac{\partial V_{h[0,0]}}{\partial \mu_{lb}} \), \( \frac{\partial V_{h[0,0]}}{\partial \mu_{h[0,0]}} \), and \( \frac{\partial V_{h[0,0]}}{\partial \mu_{cs}} \):

\[
\frac{\partial V_{h[0,0]}}{\partial \mu_{bs}} = \frac{\phi_h (r + \gamma_d + q_{lb} \phi)}{(r + \gamma_d) k^2} B
\tag{A.41}
\]

\[
\frac{\partial V_{h[0,0]}}{\partial \mu_{lb}} = \frac{\phi_h (r + \gamma_d + q_{lb} \phi)}{(r + \gamma_d) k^2} B
\tag{A.42}
\]

\[
\frac{\partial V_{h[0,0]}}{\partial \mu_{h[0,0]}} = \frac{A (r + \gamma_d + q_{lb} \phi)}{k (r + \gamma_d) \phi_h} \left( \frac{r + \gamma_d + \gamma_u + q_{cs} \phi}{C} \right),
\tag{A.43}
\]

\[
\frac{\partial V_{h[0,0]}}{\partial \mu_{cs}} = A \frac{q_{cb} (r + \gamma_d + q_{lb} \phi)}{k (r + \gamma_d) C} \left( \frac{r + \gamma_d + \gamma_u + q_{cs} \phi}{C} \right),
\tag{A.44}
\]

where

\[
B \equiv x_b + \frac{q_{cb} A}{C} \left( \frac{r + \gamma_d + q_{lb} \phi}{k} \right) - A - \left( \frac{r + \gamma_d + q_{lb} \phi}{k} \right) x_b
\tag{A.45}
\]

\[
A \equiv x_c + \frac{(x_c - 2) (q_{cs} + r + \gamma_d + \gamma_u)}{q_{cs} + r + \gamma_u} - \frac{q_{bs} x_b \phi_h}{r + \gamma_d + q_{lb} \phi_l}
\tag{A.46}
\]

\[
C \equiv \frac{q_{cb} (r + \gamma_d + q_{lb} \phi)}{k} + \frac{q_{cb} (r + \gamma_d + q_{lb} \phi)}{k}.
\tag{A.47}
\]

From here, \( \frac{\partial V_{h[0,0]}}{\partial \mu_{bs}} > 0 \) while \( \frac{\partial V_{h[0,0]}}{\partial \mu_{cs}} < 0 \), implying that the third term in (A.40) is negative. Since \( \frac{\partial V_{h[0,0]}}{\partial \mu_{lb}} < 0 \), the fourth term in (A.40) is also negative. The signs of \( \frac{\partial V_{h[0,0]}}{\partial \mu_{bs}} \) and
\[
\frac{\partial V_{h[0,0]}}{\partial q_{cb}} \text{ depend on the sign of } B:
\]
\[
B = x_b + \frac{q_{cb}}{C} \frac{(r + \gamma d + q_{cb} \phi_l)}{k} A - \frac{q_{cb}}{C} A - \frac{q_{cb}}{C} \frac{(r + \gamma d + q_{cb} \phi_l)}{k} x_b
\]
\[
= x_b \left( 1 - \frac{q_{cb}}{C} \frac{(r + \gamma d + q_{cb} \phi_l)}{k} \right) - \left( 1 - \frac{q_{cb}}{C} \frac{(r + \gamma d + q_{cb} \phi_l)}{k} \right) q_{cb} A
\]
\[
= \left( 1 - \frac{q_{cb}}{C} \frac{(r + \gamma d + q_{cb} \phi_l)}{k} \right) \left( x_b - q_{cb} A \right). \quad (A.48)
\]

First, \( 0 < \frac{q_{cb} \phi_l + \gamma d + r}{k} < 1 \) and \( 0 < \frac{q_{cb}}{C} < 1 \). To see the latter, let \( \phi_l = \phi_h \), then \( C > q_{cs} \). Since I restrict to parameter conditions such that \( \frac{\rho F_h}{\gamma_d} > S + \frac{F_i}{\gamma_u} \), we have
\[
\mu_{h[0,0]} + \mu_{h[0,1]} + \mu q_{h[1,0]} = \frac{\rho F_h}{\gamma_d} > S + \frac{F_i}{\gamma_u}. \quad (A.49)
\]

Using the CDS market clearing condition, we have \( \frac{F_i}{\gamma_u} = \mu_{0,0} + \mu_{[0,1]} = \mu_{h[0,0]} + \left( \mu_{h[0,1]} + \mu_{a[0,1]} \right) \). Consequently,
\[
\mu_{h[0,0]} + \mu_{h[0,1]} + \mu_{h[1,0]} > S + \mu_{[0,0]} + \left( \mu_{h[0,1]} + \mu_{a[0,1]} \right). \quad (A.50)
\]

Canceling \( \mu_{h[0,1]} \), we get
\[
\mu_{h[0,0]} + \mu_{h[1,0]} > S + \mu_{[0,0]} + \mu_{a[0,1]} \quad (A.51)
\]
\[
\mu_{h[0,0]} \left( S - \mu_{h[1,0]} \right) + \mu_{[0,0]} + \mu_{a[0,1]} > \mu_{[0,0]} + \mu_{a[0,1]} \quad (A.52)
\]

Hence, \( q_{cs} > q_{cb} \) and \( C > q_{cs} > q_{cb} \). Thus, the term in the first bracket of \( B \) is positive. Now consider the term in the second bracket of \( B \), \( x_b - q_{cb} A = x_b - q_{cb} \Delta h[0,1] \):
\[
\frac{x_b - q_{cb} \Delta h[0,1]}{\phi_h} = x_b - q_{cb} \frac{x_b + \frac{(x_b - 2y)(q_{cs} + x + \gamma_d + \gamma_u)}{q_{cs} + x + \gamma_d + \gamma_u} - \frac{q_{cb} \phi_h}{k} x_b}{\phi_h}
\]
\[
= x_b - \frac{x_b + \frac{(x_b - 2y)(q_{cs} + x + \gamma_d + \gamma_u)}{q_{cs} + x + \gamma_d + \gamma_u} - \frac{q_{cb} \phi_h}{k} x_b}{\phi_h}
\]
\[
= \frac{r + \gamma_d + \gamma_u + q_{cs} \phi_l + q_{cb} \phi_h}{q_{cb} \phi_h} x_b - \left( x_b + \frac{(x_b - 2y)(q_{cs} + x + \gamma_d + \gamma_u)}{q_{cs} + x + \gamma_d + \gamma_u} \right). \quad (A.53)
\]

The sign of the expression depends on the numerator:
\[
\left( \frac{r + \gamma_d + \gamma_u + q_{cs} \phi_l + q_{cb} \phi_h}{q_{cb} \phi_h} \right) x_b - \left( x_b + \frac{(x_b - 2y)(q_{cs} + x + \gamma_d + \gamma_u)}{q_{cs} + x + \gamma_d + \gamma_u} \right). \quad (A.54)
\]

This expression is positive from (9). Thus, \( \frac{\partial V_{h[0,0]}}{\partial q_{cb}} > 0 \), together with \( \frac{\partial \mu_{h[1,0]}}{\partial q_{h[0,0]}} < 0 \), implies that the first term of (A.40) is negative. Also, since \( \frac{\partial V_{h[0,0]}}{\partial q_{cb}} < 0 \), the second term of (A.40) is also negative.

Finally, from (A.37) and using the Implicit Function Theorem,
\[
\frac{\partial \mu_{h[0,0]}}{\partial \rho} = F_h \left( \frac{s \lambda_{h[0,0]} \gamma_d}{\lambda_{h[0,0]} + \gamma_d} + \frac{\lambda_{f_{h[0,0]}} \gamma_d + \gamma_u}{\lambda_{h[0,0]} + \gamma_u} + 1 \right). \quad (A.55)
\]

Thus, \( \frac{\partial \mu_{h[0,0]}}{\partial \rho} > 0 \), and, consequently, \( \frac{\partial V_{h[0,0]}(\rho)}{\partial \rho} < 0 \).

**Lemma 2. Existence**

**Proof.** To show existence, we verify that the conjectured optimal trading strategies are in fact
optimal. In particular, first, we show that the total surplus from trading the bond is positive: 
\[ \omega_b = V_{b[1,0]} - V_{b[0,0]} - V_{a[1,0]} > 0. \] 
By construction, this will ensure that individual surpluses to the buyer and the seller of the bond are positive: A high-valuation agent optimally chooses to buy the bond, and an average-valuation agent prefers to sell her bond. Second, we show that the total surplus from trading CDS is positive \( \omega_c = V_{h[0,1]} - V_{h[0,0]} + V_{l[0,1]} - V_{l[0,0]} > 0. \) This will imply that the high-valuation agents will want to sell CDS, while low-valuation agents will want to buy CDS. Third, we verify that the average-valuation agents will prefer to quit being a CDS seller: \( 0 - V_{a[0,1]} > 0. \) Thus, agents who have previously sold CDS when they were a high-valuation investor will prefer to find another seller to take over their sides of the trade and exit the market with zero utility. I proceed by first deriving \( \omega_b, \omega_c, \) and \( V_{a[0,1]}. \)

Subtracting \( rV_{l[0,0]} \) (A.25) from \( rV_{l[0,1]} \) (A.30) and defining \( \Delta_{l[0,-1]} = V_{l[0,1]} - V_{l[0,0]} \), we get

\[
\Delta_{l[0,-1]} = \frac{\delta_c + x_{cl} - y - p_c}{r + \gamma_u + q_{cs}}.
\] (A.56)

From (6),

\[
\Delta_{h[0,1]} = \frac{\phi}{1 - \phi} \Delta_{l[0,-1]} = \frac{\phi}{1 - \phi} \frac{\delta_c + x_{cl} - y - p_c}{r + \gamma_u + q_{cs}}.
\] (A.57)

Also from the value function of \( V_{a[0,1]}, \)

\[
V_{a[0,1]} = p_c - (y + \delta_c),
\] (A.58)

Substituting (A.28) into the expression for \( V_{a[0,1]}, \)

\[
rV_{h[0,1]} = p_c - (\delta_c - x_{ch}) - y + \gamma_d \left( \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cs}} - V_{h[0,1]} \right) - \gamma_u \Delta_{h[0,1]}.
\] (A.59)

Adding \( \gamma_d V_{h[0,1]} \) to both sides and subtracting \( (r + \gamma_d) V_{h[0,0]}, \)

\[
(r + \gamma_d + \gamma_u) \Delta_{h[0,1]} = p_c - (\delta_c - x_{ch}) - y + \gamma_d \left( \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cs}} - (r + \gamma_d) V_{h[0,0]} \right).
\] (A.60)

From the solution to the equations for \( V_{h[1,0]}, V_{a[1,0]}, \) and \( V_{h[0,0]}, \)

\[
V_{h[0,0]} = \frac{q_{bs} x_b \phi + \Delta_{h[0,1]} q_{cb} (r + \gamma_d + q_{bs} (1 - \phi))}{(r + \gamma_d) k},
\] (A.61)

Thus, we have three equations—(A.57), (A.60), and (A.61)—and three unknowns, \( \Delta_{h[0,1]}, p_c, \) \( V_{h[0,0]} \). The solution for \( \Delta_{h[0,1]} \) is given by

\[
\Delta_{h[0,1]} = \frac{x_{ch} + (q_{cs} + r + \gamma_u + \gamma_d) \frac{x_{cl} - 2y}{r + \gamma_u + q_{cs}} - \frac{1}{k} q_{bs} \phi x_b}{(1 - \phi) q_{cs} + r + \gamma_u + \gamma_d} + \frac{1}{k} q_{cb} (r + \gamma_d + (1 - \phi) q_{bs}).
\] (A.62)

From here,

\[
p_c = \delta_c + x_{cl} - y - \frac{1 - \phi}{\phi} (r + \gamma_u + q_{cs}) \Delta_{h[0,1]}
\] (A.63)

\[
\omega_c = \frac{1}{\phi} \Delta_{h[0,1]}
\] (A.64)

Using the solutions for \( V_{h[1,0]}, V_{a[1,0]}, \) and \( V_{h[0,0]}, \)

\[
\omega_b = \frac{x_b}{k} - \frac{q_{bs} \Delta_{h[0,1]}}{k}.
\] (A.65)
To consider small search frictions, define $\epsilon \equiv \frac{1}{\lambda_{b}}$ and $n \equiv \frac{\lambda_{c}}{\lambda_{b}}$. We show existence for $\epsilon = 0$. Then by continuity, existence is established in the neighborhood of $\epsilon = 0$ or for small search frictions. With the change of variables, (A.37) becomes:

$$
\rho F_h - \gamma_d \mu_{h[0,0]} \left( \frac{S}{\mu_{h[0,0]} + \epsilon \gamma_d} + \frac{n F_1}{\gamma_u \left( \lambda_{c} \mu_{h[0,0]} + \gamma_d + \gamma_u \right)} + 1 \right) = 0. 
$$

(A.66)

From (A.66), for any $\rho \in [0,1]$, $\mu_{h[0,0]}$ asymptotically converges to $\mu_{h[0,0]} = \frac{\rho F_h}{\gamma_d} - \left( S + \frac{F_1}{\gamma_u} \right)$. Therefore, $0 < \lim_{\lambda_{b}, \lambda_{c} \to \infty} \mu_{h[0,0]} < \infty$, and $\lim_{\lambda_{b}, \lambda_{c} \to \infty} q_{bd} = \infty$. This also implies from (A.33) that $\lim_{\lambda_{b}, \lambda_{c} \to \infty} \mu_{a[1.0]} = 0$ and $q_{bs}$ converges to a finite number. Analogously, $\lim_{\lambda_{b}, \lambda_{c} \to \infty} q_{cs} = \infty$, and, from (A.31) and (A.35), $0 < \lim_{\lambda_{b}, \lambda_{c} \to \infty} q_{cb} < \infty$.

To show $\omega_{c} > 0$ using these limits, consider the numerator of $\Delta_{h[0,1]}$:

$$
x_{ch} + (q_{cs} + r + \gamma_u + \gamma_d) \frac{x_{cl} - 2y}{r + \gamma_u + q_{cs}} - \frac{1}{k} q_{bs} \phi_{x_b}. 
$$

(A.67)

Using the above limits of $q_{cs}$, $q_{bs}$, and $q_{bd}$, it converges to $x_{ch} + x_{cl} - 2y$, which is positive by Assumption 1.

From (A.58), in order for $V_{a[1,0]} < 0$ to hold, the CDS price has to be such that $p_{c} < \delta_{c} + y$. From (A.62) and (A.63),

$$
p_{c} = (\delta_{c} + x_{cl}) - y - \frac{1}{\phi_{h}} \left( q_{cs} + r + \gamma_u \right) \frac{\left( \frac{x_{ch} + \left( x_{ch} + x_{cl} - 2y \right) q_{cs} + \gamma_d r + \gamma_u q_{cs}}{q_{cs} + r + \gamma_u} - \frac{x_{cb} \phi_{x_b}}{\phi_{h}} \right)}{\phi_{h} \left( \frac{q_{sb} \phi_{x_b} + \gamma_d r}{q_{cb}} + \frac{1 - \phi_{x_b} q_{cb} + \gamma_d r + \gamma_u q_{cs}}{\phi_{h}} \right)}.
$$

(A.68)

This converges to $\delta_{c} + y - x_{ch}$, which is less than $\delta_{c} + y$. Thus, $V_{a[1,0]} < 0$.

Average-valuation agents will not want to buy CDS because the flow utility would be $\delta_{c} - y - p_{c}$. Given that $p_{c} \to \delta_{c} + y - x_{ch}$, the flow utility, $\delta_{c} - y - p_{c}$, converges to $x_{ch} - 2y$ which is negative by Assumption (1).

To show $\omega_{b} > 0$, consider the numerator of (A.65): $x_{b} - q_{cb} \Delta_{h[0,1]}$. Since $0 < \lim q_{cb} < \infty$ and $\Delta_{h[0,1]}$ converges to zero, $x_{b} - q_{cb} \Delta_{h[0,1]}$ converges to $x_{b} > 0$. The above results show existence for $\epsilon = 0$. By continuity, existence is also established near $\epsilon = 0$.

Proof of Proposition 2. The bond price is $p_{b} = \phi(V_{b[1,0]} - V_{h[0,0]}) + (1 - \phi) V_{a[1,0]}$. Solving $V_{b[1,0]}$ and $V_{a[1,0]}$:

$$
V_{b[1,0]} = \frac{\delta_{b} + x_{b} - y}{r} - \frac{\gamma_d \left( x_{b} + q_{bb}(1 - \phi) V_{h[0,0]} \right)}{r (r + \gamma_d + q_{bb}(1 - \phi))} 
$$

(A.69)

$$
V_{a[1,0]} = \frac{\delta_{b} + x_{b} - y}{r} - \frac{(r + \gamma_d) \left( x_{b} + q_{bb}(1 - \phi) V_{h[0,0]} \right)}{r (r + \gamma_d + q_{bb}(1 - \phi))}, 
$$

(A.70)

where, from an earlier derivation,

$$
V_{b[0,0]} = \frac{q_{bb} x_{b} \phi + \Delta_{h[0,1]} q_{bb} (r + \gamma_d + q_{bb}(1 - \phi))}{(r + \gamma_d) k}. 
$$

(A.71)

Thus, we derive the limits of the $q$'s and $\Delta_{h[0,1]}$ as $\lambda_{b} \to \infty$ for an arbitrary $\lambda_{c}$. With the change of variable, $\epsilon \equiv \frac{1}{\lambda_{b}}$, (A.37) becomes

$$
\rho F_h - \gamma_d \mu_{h[0,0]} \left( \frac{S}{\mu_{h[0,0]} + \epsilon \gamma_d} + \frac{\lambda_{c} F_1}{\gamma_u \left( \lambda_{c} \mu_{h[0,0]} + \gamma_d + \gamma_u \right)} + 1 \right) = 0. 
$$

(A.72)
For $\epsilon = 0$, 
\[
\frac{\rho F_t}{\gamma_d} - S - \mu_{h[0,0]} \left( \frac{\lambda_c F_t}{\gamma_u (\lambda_c \mu_{h[0,0]} + \gamma_d + \gamma_u)} + 1 \right) = 0. \tag{A.73}
\]

For any $\rho \in [0, 1]$, the LHS of (A.73) is positive at $\mu_{h[0,0]} = 0$, decreasing in $\mu_{h[0,0]}$, and negative for large $\mu_{h[0,0]}$. Hence, (A.73) has a positive finite solution, $0 < \lim_{\lambda_b \to \infty} \mu_{h[0,0]} < \infty$, and this implies that $\lim q_{bb} = \infty$, and $k \to \infty$. In addition, $\lim_{\lambda_b \to \infty} \mu_{a[1,0]} = 0$, and $q_{bs}$ converges to a finite number from (A.33). Analogously, $\lim q_{cs} = \infty$ and, from (A.31) and (A.35), $0 < \lim q_{cb} < \infty$.

Then, as discussed above, the numerator of $\Delta h_{[0,1]}$ converges to a finite number, while the denominator converges to $\infty$, thus, $\Delta h_{[0,1]} \to 0$. So $V_{h[0,0]} \to 0$ which implies that $V_{h[1,0]} \to \frac{\delta_h + x_h - y}{r}$, $V_{a[1,0]} \to \frac{\delta_h + x_h - y}{r}$, and $p_b \to \frac{\delta_h + x_b - y}{r}$.

**Proof of Proposition 3.** Combining (A.69)–(A.71), we get the bond price.

**Proof of Proposition 4.** Consider parameter conditions such that $V_{h[0,0]}(\rho_{nocds}) = V_{h[0,0]}(\rho_{cds}) = O_b$. Since the bond price is $p_b = \phi(V_{h[1,0]} - V_{h[0,0]} + (1 - \phi)V_{a[1,0]}$, for an interior solution ($V_{h[0,0]} = V_{h[0,0]} = O_b$), it is sufficient to show that $V_{h[1,0]}(\rho_{cds}) > V_{h[1,0]}(\rho_{nocds})$ and $V_{a[1,0]}(\rho_{cds}) > V_{a[1,0]}(\rho_{nocds})$. From (A.69) and (A.70), the derivatives of $V_{h[1,0]}$ and $V_{a[1,0]}$ with respect to $q_{bb}$ are
\[
\frac{\partial V_{h[1,0]}}{\partial q_{bb}} = -\frac{\gamma_d ((r + \gamma_d) V_{h[0,0]} - x_b) (1 - \phi)}{r (r + \gamma_d + q_{bb} (1 - \phi))^2} \tag{A.74}
\]
\[
\frac{\partial V_{a[1,0]}}{\partial q_{bb}} = -\frac{(r + \gamma_d) ((r + \gamma_d) V_{h[0,0]} - x_b) (1 - \phi)}{r (r + \gamma_d + q_{bb} (1 - \phi))^2}. \tag{A.75}
\]

Thus, the condition for both $V_{h[1,0]}$ and $V_{a[1,0]}$ to be increasing in $q_{bb}$ at $q_{bb} = q_{bb}^{nocds}$ is: $(r + \gamma_d)V_{h[0,0]} - x_b < 0$ evaluated at $q_{bb} = q_{bb}^{nocds}$. The solution for $V_{h[0,0]}$ without the CDS market is
\[
V_{h[0,0]}^{nocds} = \frac{q_{bs} x_b \phi (r + \gamma_d)(r + \gamma_d + q_{bs} \phi + q_{bb} (1 - \phi)). \tag{A.76}
\]

Rearranging (A.76), we get:
\[
(r + \gamma_d) V_{h[0,0]} = \frac{q_{bs} \phi}{r + \gamma_d + q_{bs} \phi + q_{bb} (1 - \phi)} x_b < x_b. \tag{A.77}
\]

Next, we show that $q_{bb} = \lambda_b \mu_{h[0,0]}$ is greater in the presence of CDS positions. Consider the solution for $V_{h[0,0]}$ in the presence of the CDS market:
\[
V_{h[0,0]}^{cds} = \frac{x_b q_{bs}^{cds} \phi_h}{k_{cds} (\gamma_d + r)} + \frac{q_{bs} \Delta h_{[0,1]} (q_{bs} \phi_l + \gamma_d + r)}{k_{cds} (\gamma_d + r)}. \tag{A.78}
\]

Compare this with (A.76). The fact that $V_{h[0,0]}^{nocds} = V_{h[0,0]}^{cds} = O_b$ and that the second term of (A.78) is asymptotically positive implies that
\[
\frac{x_b q_{bs}^{cds} \phi_h}{k_{cds} (\gamma_d + r)} < \frac{x_b q_{bs} \phi_h}{k (\gamma_d + r)}. \tag{A.79}
\]

The term, $\frac{x_b q_{bs} \phi_h}{k_{(\gamma_d + r)}}$, is strictly decreasing in $\mu_{h[0,0]}$. Thus, $\mu_{h[0,0]}^{cds} > \mu_{h[0,0]}^{nocds}$.

Now consider the effect on the bond bid-ask spread, $\omega_b$, characterized in (A.65). The first term, $x_b/k$, in (A.65) is strictly decreasing in $\mu_{h[0,0]}$, while the second term is asymptotically positive and arises due to CDS. Thus, $\omega_b^{cds} < \omega_b^{nocds}$.
Now consider bond volume, $\lambda_b\mu_{h[0,0]/\mu_{a[1,0]}}$. Using the expression for $\mu_{a[1,0]}$ from (A.33), and taking the derivative with respect to $\mu_{h[0,0]}$, we get
\[
\frac{\lambda_b \gamma_d^2}{(\gamma_d + \lambda_b \mu_{h[0,0]})^2} > 0. \tag{80}
\]
Thus, bond volume is increasing in $\mu_{h[0,0]}$, and, hence, bond volume is greater with the existence of the CDS market.

\[\square\]

**Proof of Proposition 5.** For an arbitrary $\lambda_b$, consider what (A.37) limits to as $\lambda_c \to \infty$:
\[
\frac{\rho F_h}{\gamma_d} - \left( \frac{S\lambda_b \mu_{h[0,0]} + F_l}{\lambda_b \mu_{h[0,0]} + \gamma_d \mu_{h[0,0]} + \mu_{h[0,0]}} \right) = 0. \tag{81}
\]
The LHS of (81) is positive at $\mu_{h[0,0]} = 0$, decreasing in $\mu_{h[0,0]}$, and negative for large $\mu_{h[0,0]}$. Thus, for any $\rho$, $\mu_{h[0,0]}$ is finite as $\lambda_c \to \infty$. As a result, $\mu_{a[1,0]}$, $q_{bs}$ and $q_{bb}$ are finite. Since $\mu_{t[0,0]} + \mu_{a[1,0]} \to 0$, $q_{dt}$ is also finite. But $q_{cs} = \lambda_c \mu_{h[0,0]} \to \infty$. Thus, $\Delta_{h[0,1]} \to 0$.

When the solution is interior,
\[
V_{h[0,0]}^{cds} = V_{h[0,0]}^{nocds} = O_h. \tag{82}
\]
Then, using $\Delta_{h[0,1]} \to 0$ and (A.61):
\[
\frac{x_b q_{bs}^{cds} \phi_h}{k^{cds} (\gamma_d + r)} = \frac{x_b q_{bs}^{nocds} \phi_h}{k^{nocds} (\gamma_d + r)}. \tag{83}
\]
Since this expression is uniquely determined by $\mu_{h[0,0]}$,
\[
\mu_{h[0,0]}^{cds} = \mu_{h[0,0]}^{nocds}. \tag{84}
\]
As a result, $q_{bb} = \lambda_b \mu_{h[0,0]}$ is the same as without the CDS market. Consequently, from (A.69)–(A.70) and (A.82), $V_{h[0,1]}$ and $V_{a[1,0]}$ are the same with or without the CDS market. Thus, when $\lambda_c \to \infty$, the bond price is the same as in the benchmark environment without CDS. For (84) to hold, from (81), the entry rate (hence, the measure of high-valuation investors) increases enough to exactly offset the total measure of low-valuation investors $\frac{F_l}{\gamma_d}$, $\frac{(\rho_{cds} - \rho_{nocds}) F_h}{\gamma_u} = \frac{F_l}{\gamma_u}$.

If entry is exogenous, $\lim_{\lambda_c \to \infty} p_h(\lambda_c) < p_h^{nocds}$ because the measure of high-valuation investors (hence, the measure of bond buyers) decreases due the existence of low-valuation investors.

\[\square\]

**Proof of Proposition 6.** The population measures evolve according to
\[
\dot{\mu}_{h[0,0]}(t) = \rho F_h + \gamma d H_{h[0,0]}(t) - \left( \gamma d H_{h[0,0]}(t) + (q_{bs} + q_{cs}) \mu_{h[0,0]}(t) \right) \mu_{h[0,0]}(t), \tag{85}
\]
\[
\dot{\mu}_{h[1,0]}(t) = \mu_{h[0,0]}(t) - \left( \gamma d H_{h[1,0]}(t) + q_{bs} H_{a[0,0]}(t) \right) \mu_{h[1,0]}(t). \tag{86}
\]
\[
\dot{\mu}_{a[1,0]}(t) = \gamma d H_{a[1,0]}(t) - \left( \gamma d H_{a[1,0]}(t) + (q_{bs} + q_{cs}) \mu_{a[1,0]}(t) \right) \mu_{a[1,0]}(t). \tag{87}
\]
\[
\dot{\mu}_{h[0,1]}(t) = \mu_{h[0,0]}(t) - \left( \gamma d H_{h[0,1]}(t) + q_{bs} H_{a[0,1]}(t) \right) \mu_{h[0,1]}(t). \tag{88}
\]
\[
\dot{\mu}_{a[0,1]}(t) = \gamma d H_{a[0,1]}(t) - \left( \gamma d H_{a[0,1]}(t) + (q_{bs} + q_{cs}) \mu_{a[0,1]}(t) \right) \mu_{a[0,1]}(t). \tag{89}
\]
\[
\dot{\mu}_{h[0,-1]}(t) = (q_{cs} - q_{dt}) \mu_{h[0,0]}(t) - \mu_{h[0,-1]}(t). \tag{90}
\]
Value functions evolve according to

\[ V_{h[0,0]}(t) = V_{h[0,0]}(t) - \left[ \gamma_d(0 - V_{h[0,0]}(t)) + q_b(t)\phi(t) + q_c(t)(V_{h[0,1]}(t) - V_{h[0,0]}(t)) \right] \] (A.92)

\[ V_{l[0,0]}(t) = V_{l[0,0]}(t) - \left[ \gamma_a(0 - V_{l[0,0]}(t)) + q_c(t)(V_{l[0,1]}(t) - V_{l[0,0]}(t)) \right] \] (A.93)

\[ V_{h[1,0]}(t) = V_{h[1,0]}(t) - \left[ \delta_b + x_b - y + \gamma_d(V_{a[1,0]}(t) - V_{h[1,0]}(t)) \right] \] (A.94)

\[ V_{a[1,0]}(t) = V_{a[1,0]}(t) - \left[ \delta_b - y + q_b(t)(1 - \phi)\omega_b(t) \right] \] (A.95)

\[ V_{h[0,1]}(t) = V_{h[0,1]}(t) - \left[ p_c(t) - (\delta_c - x_{cl}) - y + \gamma_d(V_{a[0,1]}(t) - V_{h[0,1]}(t)) + \gamma_u(V_{h[0,0]}(t) - V_{h[0,1]}(t)) \right] \] (A.96)

\[ V_{a[0,1]}(t) = V_{a[0,1]}(t) - \left[ p_c(t) - \delta_c - y + q_c(t)(0 - V_{a[0,1]}(t)) + \gamma_a(0 - V_{a[0,1]}(t)) \right] \] (A.97)

\[ V_{l[0,1]}(t) = V_{l[0,1]}(t) - \left[ -p_c(t) + (\delta_c + x_{ch}) - y + \gamma_u(0 - V_{l[0,1]}(t)) \right] . \] (A.98)

Bond and CDS prices are given by

\[ p_b(t) = \phi V_{a[1,0]}(t) + (1 - \phi)(V_{h[1,0]}(t) - V_{h[0,0]}(t)) \] (A.99)

and

\[ V_{h[0,1]}(t) - V_{h[0,0]}(t) = \phi (V_{l[0,1]}(t) - V_{l[0,0]}(t)) + V_{h[0,1]}(t) - V_{h[0,0]}(t) . \] (A.100)

Using the ODE for \( V_{h[1,0]} \) and \( V_{h[0,0]} \),

\[ \Delta_{h[1,0]}(t) = r \Delta_{h[1,0]}(t) - \left[ \delta_b + x_b - y - (\gamma_d + q_b(t)\phi)\omega_b(t) - q_c(t)\phi\omega_c(t) \right] . \] (A.101)

Together with the ODE for \( V_{a[1,0]} \),

\[ \dot{\omega}_b(t) = -x_b + (r + \gamma_d + q_b(t)\phi + q_b(t)(1 - \phi))\omega_b(t) + q_c(t)\phi\omega_c(t) . \] (A.102)

Analogously, we get the ODE for \( \omega_c \):

\[ \dot{\omega}_c(t) = -x_{cd} + q_b(t)\phi\omega_b(t) + (r + \gamma_d + \gamma_u + \phi q_c(t) + (1 - \phi)q_c(t))\omega_c(t) \] (A.103)

To solve for \( \omega_b \) and \( \omega_c \), we write (A.102) and (A.103) as

\[ \begin{bmatrix} \dot{\omega}_b(t) \\ \dot{\omega}_c(t) \end{bmatrix} = - \begin{bmatrix} x_b \\ x_{cd} + x_{ch} - 2y \end{bmatrix} + A(t) \begin{bmatrix} \omega_b(t) \\ \omega_c(t) \end{bmatrix} , \] (A.104)

where

\[ A(t) = \begin{bmatrix} r + \gamma_d + q_b(t)\phi + q_b(t)(1 - \phi) & q_c(t)\phi \\ q_b(t)\phi & \gamma_d + \gamma_u + \phi q_c(t)(1 - \phi) \end{bmatrix} . \] (A.105)

Thus, the solution is

\[ \begin{bmatrix} \omega_b(t) \\ \omega_c(t) \end{bmatrix} = \int_t^\infty e^{-\int_u^s A(u)du} \begin{bmatrix} x_b \\ x_{cd} + x_{ch} - 2y \end{bmatrix} ds . \] (A.106)

From here, the solutions to the ODE for \( \Delta_{h[1,0]} \) and \( V_{a[1,0]} \) are given by

\[ \Delta_{h[1,0]}(t) = \frac{\delta_b + x_b - y}{r} - \int_t^\infty e^{-r(s-t)} ((\gamma_d + q_b(t)\phi)\omega_b(t) + q_c(t)\phi\omega_c(t)) ds \] (A.107)

\[ V_{a[1,0]}(t) = \frac{\delta_b - y}{r} + \int_t^\infty e^{-r(s-t)} q_b(t)(1 - \phi)\omega_b(t) ds . \] (A.108)

\( \square \)


\textbf{Proof of Propositions 7 & 8.} The characterization of the CDS price in an environment with and without frictions come as a corollary of Lemma 2 and are shown in the proof of Lemma 2. \hfill \square

\section{Covered CDS}

\textbf{Population Measures}

\begin{table}[h]
\centering
\caption{Population Measures of Investors in an Environment with Covered CDS}
\label{tab:population_measures}
\begin{tabular}{ll}
\hline
Type & Flow-in = Flow-out \\
\hline
$h[0,0]$ & $\rho F_h + \gamma_u \mu_h[0,1][0,1] + q_h \mu_h[1,0][0,1] = \gamma_d \mu_h[0,0] + (q_s + q_c) \mu_h[0,0]$ \\
$l[0,0]$ & $F_l = \gamma_u \mu_l[0,0] + q_h \mu_l[0,0]$ \\
$h[1,0]$ & $q_h \mu_h[0,0] = \gamma_d \mu_h[1,0]$ \\
a$[0,1]$ & $\gamma_d \mu_h[1,0] = (q_s + q_c) \mu_h[0,1]$ \\
h$[0,1][0,1]$ & $\lambda_h \left( \mu_h[0,0] + \mu_h[0,1][0,1] \right) \mu_h[0,0] = (\gamma_d + \gamma_u) \mu_h[0,1][0,1]$ \\
a$[0,1][0,1]$ & $\gamma_d \mu_h[0,1][0,1] = (q_s + \gamma_u) \mu_h[0,1][0,1] + q_h \mu_h[0,1][0,1]$ \\
h$[0,1][1,1]$ & $\lambda_h \left( \mu_h[0,1][0,1] + \mu_h[0,1][1,1] \right) \mu_h[0,0] = (\gamma_d + q_h) \mu_h[0,1][1,1]$ \\
a$[0,1][1,1]$ & $\gamma_d \mu_h[0,1][1,1] = q_c \mu_h[0,1][1,1] + q_h \mu_h[0,1][1,1]$ \\
l$[0,1]$ & $q_c \mu_h[0,0] = \gamma_u \mu_l[0,1]$ \\
a$[1,-1]$ & $q_c \mu_h[1,0] = q_h \mu_h[1,1]$ \\
\hline
\end{tabular}

\end{table}

Market clearing conditions are

\begin{equation}
\mu_h[1,0] + \mu_h[0,1] + \mu_a[1,1] = S \tag{B.1}
\end{equation}

\begin{equation}
\mu_h[0,1][0,1] + \mu_h[0,0][0,1] + \mu_h[0,1][1,1] + \mu_a[1,1][0,1] = \mu_a[1,1] + \mu_l[0,1]. \tag{B.2}
\end{equation}

\textbf{Value Functions}

The flow benefit of holding a covered CDS position can be seen in (B.7).

\begin{equation}
\begin{aligned}
rV_h[0,0] &= \gamma_d(0 - V_h[0,0]) + \lambda_h \mu_a[0,1][0,1](V_h[0,0] - p_h - V_h[0,0]) + \lambda_h \mu_a[1,1][0,1](V_h[1,0] - p_h[1,1] - V_h[0,0]) \tag{B.3} \\
&\quad + \lambda_h \mu_a[0,1][1,1](V_h[0,1][1,1] - V_h[0,0]) + \lambda_a(\mu_a[0,1][0,1] + \mu_a[0,1][1,1])(V_h[0,1][0,1] - V_h[0,0]).
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
rV_l[0,0] &= \gamma_u(0 - V_l[0,0]) + q_c(V_l[0,1][0,0] - V_l[0,0]) \tag{B.4} \\
rV_h[1,0] &= \delta_b + x_h - y + \gamma_d(V_a[0,1][0,1] - V_h[0,1][0,1]) \tag{B.5} \\
rV_a[0,0] &= \delta_b + \gamma_d(V_a[1,1][0,1] - V_a[0,1][0,1] + q_h(0 - V_a[1,1][0,1] + p_h)) \tag{B.6} \\
rV_a[1,1] &= \delta_b - y + \delta_c - \mu_a[0,1][1,1] - \mu_a[1,0][1,1] + q_h(0 - V_a[0,1][1,1] + p_h) \tag{B.7} \\
rV_a[0,1][0,1] &= \delta_c - x_h - y + \gamma_d(V_a[0,0][0,1] - V_a[0,1][0,1] + q_h(0 - V_a[0,0][0,1])) \tag{B.8} \\
rV_a[0,1][1,1] &= \delta_c - \mu_a[0,1][1,1] + q_h(0 - V_a[0,0][1,1] + p_h) \tag{B.9} \\
rV_a[0,1][0,1] &= \delta_c - \mu_a[0,1][1,1] + q_h(0 - V_a[0,0][0,1] + p_h) \tag{B.10} \\
rV_a[0,1][1,1] &= \delta_c - \mu_a[0,1][1,1] + q_h(0 - V_a[0,0][1,1] + p_h) \tag{B.11} \\
rV_l[0,1] &= \delta_c + x_d - y + \gamma_u(0 - V_l[0,0]) \tag{B.12}
\end{aligned}
\end{equation}
C A Micro Foundation for Hedging Benefits

In this appendix, I derive in a CARA setting a micro foundation for the liquidity shocks $x_b$ and $x_c$ and the cost of the risk-bearing parameter, $y$. I simplify the notation by denoting the continuous time dependence $y(t)$ as $y_t$.

Agents have CARA utility preferences with risk aversion $\alpha$: $u(c) = -\exp(-\alpha c)$ and time preference rate $\beta$. The cash flow of the bond is given by a cumulative dividend process, $D^b_t$:

$$dD^b_t = \left(\delta^b dt + \sigma^b dB_t\right).$$  \hfill(C.1)

where $B_t$ is a standard Brownian motion, and $\delta^b > 0$ and $\sigma^b > 0$ are constants. The CDS buyer’s cash flow is given by the process $D^c_t$:

$$dD^c_t = (\delta^c dt - \sigma^c dB_t).$$  \hfill(C.2)

Thus, $D^b_t$ and $D^c_t$ are perfectly negatively correlated.

Agents have an idiosyncratic cumulative endowment process:

$$de_t = \sigma_e \left[ \rho_t dB_t + \sqrt{1 - \rho_t^2} dZ_t \right],$$  \hfill(C.3)

where $\sigma_e > 0$ is a constant, $Z_t$ is a standard Brownian motion independent of $B_t$, and $\rho_t$ is the instantaneous correlation process between the bond cash flow and agents’ endowment process. The correlation process, $\rho_t$, is a three-state Markov chain with states $\rho_t \in \{\rho_l, 0, \rho_h\}$, where $\rho_l > 0 > \rho_h$. If $\rho_t = \rho_l$, the agent is currently a low-valuation investor, and her endowment process is positively correlated with the bond’s cash flow, $D^b_t$. If $\rho_t = 0$, an agent’s endowment process has no correlation with the bond cash flow. If $\rho_t = \rho_h < 0$, an agent is a high-valuation type as her endowment process is negatively correlated with the bond cash flow. (Hence, she would be more willing to be exposed to the bond relative to a low- or average-valuation investor.) Analogous to the baseline model, a low-valuation agent switches to a high-valuation with intensity $\gamma_u$, and a high-valuation switches to a low-valuation with intensity $\gamma_u$. For an average-valuation agent, the intensity of switching to either a high- or a low-valuation is zero.

We restrict the agent’s asset position in the bond market to $\theta^b_t \in \{0, 1\}$ and in the CDS market to $\theta^c_t \in \{-1, 0, 1\}$. The set of agent types $T$ is the same as in the baseline model.

An agent’s optimization problem is

$$J(W_0, \tau_0) = \max_{\{c_t\}} \mathbb{E} \left[ \int_0^\infty e^{-\beta t} u(c_t) dt \right]$$  \hfill(C.4)

subject to

$$dW_t = (rW_t - c_t) dt + de_t + dD^b_t \theta^b_t - p_b d\theta^b_t + (p_c dt - dD^c_t) \theta^c_t,$$  \hfill(C.5)

where $W_t$ is the agent’s wealth process, $W_0$ is given, $p_b$ is the bond price, and $p_c$ is the CDS price.
Deriving the Hamilton-Jacobi-Bellman Equation

Next, we derive the Hamilton-Jacobi-Bellman (HJB) equation. Equation (C.4) can be written recursively as

\[ J(W_t, \tau_t) = \max_c u(c_t) \Delta t + (1 - \beta \Delta t) E J(W_{t+\Delta t}, \tau_{t+\Delta t}). \]  

(C.6)

Subtract \((1 - \beta \Delta t) J(W_t, \tau_t)\) from both sides and divide by \(\Delta t\) to get

\[ \beta J(W_t, \tau_t) = \max_c u(c_t) + (1 - \beta \Delta t) E \left[ \frac{J(W_{t+\Delta t}, \tau_{t+\Delta t}) - J(W_t, \tau_t)}{\Delta t} \right]. \]  

(C.7)

As \(\Delta t \to 0\),

\[ \beta J(W_t, \tau_t) = \max_c u(c_t) + E \left[ \frac{dJ(W_t, \tau_t)}{dt} \right]. \]  

(C.8)

The next step is deriving the expectation of the total differential of \(J(W_t, \tau_t)\). Approximating the total differential \(dJ(W_t, \tau_t)\) with a Taylor-series expansion and taking its expectation, we get

\[ EdJ(W_t, \tau_t) = J_W(W_t, \tau_t)E[dw_t] + \frac{1}{2} J_{WW}(W_t, \tau_t)E[dw_t^2] + E[j_r(W_t, \tau_t)d\tau_t]. \]  

(C.9)

Using the expressions for the bond and CDS cash flows and the endowment process, \(E[dw_t]\) and \(E[dw_t^2]\) are given by

\[ E[dw_t] = \left( rW_t - c_t + \delta b_t + (p_c - \delta^c)\theta_t^c \right) dt \]  

(C.10)

\[ E[dw_t^2] = \left( \sigma_D^b \theta_t^b + \sigma_D^\theta \theta_t^\theta \right)^2 + 2\sigma_e \rho_t \left( \sigma_D^b \theta_t^b + \sigma_D^\theta \theta_t^\theta \right) + \sigma_e^2 \]  

(C.11)

Substituting the above expressions for \(E[dw_t]\) and \(E[dw_t^2]\) back into \(EdJ(W_t, \tau_t)\), we get

\[ EdJ(W_t, \tau_t) = J_W(W_t, \tau_t) \left[ \left( rW_t - c_t + \delta b_t + (p_c - \delta^c)\theta_t^c \right) dt \right] + E[j_r(W_t, \tau_t)d\tau_t] \]  

(C.12)

\[ + \frac{1}{2} J_{WW}(W_t, \tau_t) \left( \sigma_D^b \theta_t^b + \sigma_D^\theta \theta_t^\theta \right)^2 + 2\sigma_e \rho_t \left( \sigma_D^b \theta_t^b + \sigma_D^\theta \theta_t^\theta \right) + \sigma_e^2 \]  

\[ + E[j_r(W_t, \tau_t)d\tau_t]. \]

I consider the steady state. Substituting (C.12) into (C.8), the HJB is given by

\[ \beta J(W, \tau) = \max_c u(c) + J_W(W, \tau) \left[ rW - c + \delta b + (p_c - \delta^c)\theta_c^c \right] \]  

(C.13)

\[ + \frac{1}{2} J_{WW}(W, \tau) \left( \sigma_D^b \theta_b + \sigma_D^\theta \theta_c \right)^2 + 2\sigma_e \rho_t \left( \sigma_D^b \theta_b + \sigma_D^\theta \theta_c \right) + \sigma_e^2 \]  

\[ + E[j_r(W_t, \tau_t)d\tau_t]. \]

Proposition 9. Let \(P(\tau, \tau')\) be the instantaneous payoff associated with a transition of a type \(\tau\) investor to type \(\tau'\), and let \(\gamma(\tau', \tau)\) denote the intensity of switching from type \(\tau\) to type \(\tau'\). The solutions to the HJB equations, \(J(W, \tau)\), are

\[ J(W, \tau) = -e^{-r\alpha(W_t+V_t+\bar{a})}, \]  

(C.14)
where \( \bar{a} = \frac{1}{r} \left( \frac{\log(r)}{\alpha} - \frac{r - \beta}{r\alpha} - \frac{r}{2}r\alpha\sigma_e^2 \right) \). The constant \( V_r \) is given by

\[
rV_r = \left( \delta^b - x_b(\tau) \right) \theta_b - y(\tau) \left( \theta_b \right)^2 + \left( p_c - \left( \delta^c + x_c(\tau) \right) \right) \theta_c - y(\tau) \left( \theta_c \right)^2 + \frac{r}{r\alpha} \left( 1 - e^{-r\alpha(V_r - V_0 + P(\tau, \tau'))} \right),
\]

where \( x_b(\tau) = r\alpha\rho_r \sigma_e \alpha_D^b, \ x_c(\tau) = r\alpha\rho_r \sigma_e \alpha_D^c, \) and \( y(\tau) = \frac{r\alpha}{\sigma} \left( \sigma(\tau) \right)^2 \).

**Proof.** Using the guessed functional form, \( J(W, \tau) = -e^{-r\alpha(W_0 + V_0 + \bar{a})} \), and the first order condition of (C.13), we can solve for the optimal consumption rate for agent \( \tau \):\(^{35}\)

\[
c_r = -\frac{\log(r)}{\alpha} + r \left( W + V_x + \bar{a} \right).
\]

Inserting the optimal consumption back into (C.13) and using \( J(W, \tau) = -e^{-r\alpha(W_0 + V_0 + \bar{a})} \), \( J_W = r\alpha e^{-r\alpha(W_0 + V_0 + \bar{a})} \) and \( J_{WW} = -r^2\alpha^2 e^{-r\alpha(W_0 + V_0 + \bar{a})} \), we get

\[
-\beta e^{-r\alpha(W_0 + V_0 + \bar{a})} = -e^{\log(r) - r\alpha(W_0 + V_0 + \bar{a})} \left( \frac{\log(r)}{\alpha} - r \left( V_0 + \bar{a} \right) + \delta^b \theta_b + \left( p_c - \delta^c \right) \theta_c \right)
\]

\[
- \frac{1}{2} \left[ \frac{r\alpha}{\alpha(r\alpha - 1) - \frac{1}{2}r\alpha\sigma_e^2} \right] \left( \left( \sigma_D^b \theta_b + \sigma_D^c \theta_c \right)^2 + 2\sigma_e \rho_r \left( \sigma_D^b \theta_b + \sigma_D^c \theta_c \right) \right)
\]

\[
+ \frac{E \left[ J_r(W, \tau) d\tau \right]}{\alpha(r\alpha - 1) - \frac{1}{2}r\alpha\sigma_e^2}.
\]

Dividing both sides by \( -\frac{1}{r\alpha} e^{-r\alpha(W_0 + V_0 + \bar{a})} \) and rearranging, we get

\[
0 = rV_r - e^{r\alpha(W_0 + V_0 + \bar{a})} \frac{E \left[ J_r(W, \tau) d\tau \right]}{r\alpha} + r\bar{a} - \frac{1}{r} \left[ \frac{\log(r)}{\alpha} - r - \frac{\beta}{\alpha} - \frac{1}{2}r\alpha\sigma_e^2 \right] + \left( \delta^b \theta_b - \frac{1}{2}r\alpha \left( \left( \sigma_D^b \theta_b + \sigma_D^c \theta_c \right)^2 + 2\sigma_e \rho_r \left( \sigma_D^b \theta_b + \sigma_D^c \theta_c \right) \right) + \left( p_c - \delta^c \right) \theta_c \right).
\]

Defining \( \bar{a} \equiv \frac{1}{r} \left( \frac{\log(r)}{\alpha} - \frac{r - \beta}{r\alpha} - \frac{1}{2}r\alpha\sigma_e^2 \right) \) and using \( \theta^b \theta^c = 0 \),

\[
rV_r = \left( \delta^b - r\alpha \sigma_e \rho_r \sigma_D^b \right) \theta_b - \frac{1}{2}r\alpha \left( \left( \sigma_D^b \theta_b \right)^2 + \left( \sigma_D^c \theta_c \right)^2 \right) \left( p_c - \delta^c - r\alpha \sigma_e \rho_r \sigma_D^c \right) \theta_c
\]

\[
+ e^{r\alpha(W_0 + V_0 + \bar{a})} \frac{E \left[ J_r(W, \tau) d\tau \right]}{r\alpha}.
\]

Define \( x_b(\tau) = r\alpha \sigma_e \rho_r \sigma_D^b, \ x_c(\tau) = r\alpha \sigma_e \rho_r \sigma_D^c, \ y(\tau) = \frac{r\alpha}{\sigma} \left( \sigma(\tau) \right)^2, \) and \( \sigma(\tau) = \sigma_D^b \left( \theta_b(\tau) \right)^2 + \sigma_D^c \left( \theta_c(\tau) \right)^2 \). Given these definitions of \( x_b(\tau), x_c(\tau), \) and \( y(\tau) \),

\[
rV_r = \left( \delta^b - x_b(\tau) \right) \theta_b - y(\theta_b)^2 + \left( p_c - \left( \delta^c + x_c(\tau) \right) \right) \theta_c - y(\theta_c)^2 + e^{r\alpha(W_0 + V_0 + \bar{a})} \frac{1}{r\alpha} \frac{E \left[ J_r(W, \tau) d\tau \right]}{dt}.
\]

Consider, for example, how (C.20) looks for the \( \tau = h[0, 0] \) type. \( EJ_r(W, \tau) d\tau \) for \( \tau = h[0, 0] \) is

\[
\frac{EJ_r(W, \tau) d\tau}{dt} = \gamma_{1d} dt \left( J(W, \infty) - J(W, h[0, 0]) \right) + q_{1b} dt \left( J(W - p_b \left( \theta_c \theta_c - h[1, 0] \right) - J(W, h[0, 0]) \right)
\]

\[
+ q_{1d} dt \left( J(W, h[0, 1]) - J(W, h[0, 0]) \right).
\]

\(^{35}\)The F.O.C. with respect to \( c_r \) is: \( 0 = \alpha e^{-\alpha c} - J_W(W_x, \tau) \). Using \( J_W = r\alpha e^{-r\alpha(W_0 + V_0 + \bar{a})} \), \( e^{-r\alpha(W_0 + V_0 + \bar{a})} = e^{-\alpha c} \). Rewrite it as \( e^{\log(r)e^{-r\alpha(W_0 + V_0 + \bar{a})}} = e^{-\alpha c} \)
Using \( J(W, \tau) = -e^{-r \alpha (W_t + V_t + a)} \) and multiplying both sides by \( e^{r \alpha (W_t + V_t + a)} \), we get

\[
e^{r \alpha (W_t + V_t + a)} \frac{E[J_r(W, \tau) d\tau]}{dt} = \gamma_d \left( 1 - e^{-r \alpha (V_{\infty} - V_t)} \right) + q_{bs} \left( 1 - e^{-r \alpha (-p_b + V_{h_{[1,0]}} - V_t)} \right) + q_{cb} \left( 1 - e^{-r \alpha (V_{h_{[0,1]} - V_t})} \right).
\]

(C.22)

Thus, using the fact that \( \theta_c = \theta_b = 0 \) for the \( \tau = h_{[0,0]} \) type, (C.20) for \( \tau = h_{[0,0]} \) type is given by

\[
r V_{\tau} = \gamma_d \left( 1 - e^{-r \alpha (V_{\infty} - V_t)} \right) + q_{bs} \left( 1 - e^{-r \alpha (-p_b + V_{h_{[1,0]}} - V_t)} \right) + q_{cb} \left( 1 - e^{-r \alpha (V_{h_{[0,1]} - V_t})} \right).
\]

(C.23)

It is analogous for the other types. Table (5) gives all the possible switching intensities \( \gamma(\tau', \tau) \) from \( \tau \in \mathcal{T} \) to \( \tau' \in \mathcal{T} \).

<table>
<thead>
<tr>
<th>( \tau' )</th>
<th>( h_{[0,0]} )</th>
<th>( l_{[0,0]} )</th>
<th>( h_{[1,0]} )</th>
<th>( a_{[1,0]} )</th>
<th>( h_{[0,1]} )</th>
<th>( a_{[0,1]} )</th>
<th>( l_{[0,-1]} )</th>
<th>( \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{[0,0]} )</td>
<td>0</td>
<td>( q_{bs} )</td>
<td>0</td>
<td>( q_{d} )</td>
<td>0</td>
<td>0</td>
<td>( \gamma_d )</td>
<td></td>
</tr>
<tr>
<td>( l_{[0,0]} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( q_{cs} )</td>
<td>( \gamma_u )</td>
<td></td>
</tr>
<tr>
<td>( h_{[1,0]} )</td>
<td>0</td>
<td>0</td>
<td>( \gamma_d )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( q_{bb} )</td>
<td></td>
</tr>
<tr>
<td>( a_{[1,0]} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \gamma_u )</td>
<td>0</td>
<td>( q_{cs} )</td>
<td></td>
</tr>
<tr>
<td>( h_{[0,1]} )</td>
<td>( \gamma_u )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \gamma_u )</td>
<td>0</td>
<td>( q_{cs} )</td>
<td></td>
</tr>
<tr>
<td>( a_{[0,1]} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \gamma_u )</td>
<td>( q_{cs} )</td>
<td></td>
</tr>
<tr>
<td>( l_{[0,-1]} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \gamma_u )</td>
<td>0</td>
<td>( \gamma_u )</td>
<td></td>
</tr>
</tbody>
</table>

The instantaneous payoff, mentioned in Proposition 9, is plus and minus the bond price for sell and buy positions, respectively. The instantaneous payoff for a CDS transaction is zero as the CDS premium is paid continually and is, hence, imbedded in \( V_{\tau} \). Thus, \( P(\tau, \tau') \) is

\[
P(\tau, \tau') = \begin{cases} -p_b & \text{if } \tau = h_{[0,0]}, \ \tau' = h_{[1,0]} \\ p_b & \text{if } \tau = a_{[1,0]}, \ \tau' = h_{[1,0]} \\ 0 & \text{otherwise.} \end{cases}
\]

(C.24)

\[\square\]

**Comparison to the Baseline Model**

In the limit as \( \alpha \to 0 \), the general value function, (C.23), satisfies the value functions with risk-neutral agents of the paper’s baseline model. To see this, linearizing (C.23) using \( e^{z} - 1 \approx z \) for small \( \alpha \), we get

\[
r V_{h_{[0,0]}} = \gamma_d \left( 0 - V_{h_{[0,0]}} \right) + q_{bs} \left( -p_b + V_{h_{[1,0]}} - V_{h_{[0,0]}} \right) + q_{cb} \left( V_{h_{[0,1]} - V_{h_{[0,0]}}} \right).
\]

(C.25)

Equation (C.25) is analogous to the value functions of the baseline model with risk-neutral agents.

The baseline model is essentially a reduced form approximation of a more general specification with risk averse agents and risky assets. The hedging benefits, \( x_b(\tau) = r \alpha \rho_s \sigma_s \sigma_s^b \) and \( x_c(\tau) = r \alpha \rho_s \sigma_s \sigma_s^c \), increase with agents’ risk aversion (\( \alpha \)), bond cash flow risk (\( \sigma_s \)), endowment volatility (\( \sigma_e \)), and the correlation between the bond cash flow and the endowment process (\( \rho_r \)). The parameter \( x_{ch} \) from the baseline model is equivalent to \( x_{ch} = -x_c(\tau) = -r \alpha \rho_s \sigma_s \sigma_s^c \) if \( \tau \) is a high-valuation investor, while \( x_{cl} = r \alpha \rho_s \sigma_s \sigma_s^c \) for a low-valuation investor. Under a special case
where $\sigma^c_D = \sigma^b_D = \sigma_D$, $x_b(\tau) = x_c(\tau)$ and $y(\tau) = \frac{r_\alpha}{\tau} \left( (\sigma^b_D)^2 \right)^2 = \frac{r_\alpha}{\tau} \left( (\sigma^c_D)^2 \right)^2$ for all $\tau$. In this case, $x_b = x_{ch}$, while $x_{cl}$ is not necessarily equal to $x_{ch}$. If, in addition, $-\rho_h = \rho_l$, then $x_b = x_{ch} = x_{cl}$.

D Model Figures

Figure 2: The Effect of Naked CDS Purchases on Bond Market Liquidity

The figures compare bond market liquidity variables with CDS (solid lines) and without CDS (dashed lines) as a function of the CDS market efficiency ($\lambda_c$). Although it cannot be directly seen from the plots, the variables converge to the no-CDS environment as the CDS market frictions decrease ($\lambda_c \to \infty$). The relevant discussion is in Section 2.1, and the parameter values used to generate the plot are in Table 6.

Figure 3: The Effect of Naked CDS Purchases on Bond Market Composition

The plots compare the composition of buyers and sellers in the bond market with (solid lines) and without CDS (dashed lines) as a function of the CDS market efficiency ($\lambda_c$). Although it cannot be directly seen from the plots, the variables converge to the no-CDS environment as the CDS market frictions decrease ($\lambda_c \to \infty$). The relevant discussion is in Section 2.1, and the parameter values used to generate the plot are in Table 6.
Figure 4: The Short-Run Dynamics of Population Measures after a Temporary CDS Ban
The figures plot the time varying equilibrium path of the number of naked CDS buyers (the left panel),
bond buyers (the middle panel), and bond sellers (the right panel) from a temporary ban back to the
steady state. A temporary naked CDS ban is modeled as a shock to the steady at time $t = 0$ that sets
the number of naked CDS buyers to zero (as can be seen in the left panel). The relevant discussion is in
Section 2.2, and the parameter values used to generate the plot are in Table 6.

No. of CDS Buyers ($\mu_{b(0,0)}$)  No. of Bond Buyers ($\mu_{b(0,0)}$)  No. of Bond Sellers ($\mu_{d(0,0)}$)

Figure 5: Bond Market Liquidity Short-Run Dynamics after a Temporary CDS Ban
The figure plots the short-run dynamics of the illiquidity discount, the bid-ask spread, and the trading
volume from a temporary CDS ban back to the steady state. As described in Section 2.2, a temporary
naked CDS ban is modeled as a shock to the steady at time $t = 0$ that sets the number of naked CDS
buyers to zero. The parameter values used to generate the plot are in Table 6.

Bond Illiquidity Discount ($d_b$)  Bond Bid–Ask Spread ($\omega_b/p_b, %$)  Bond Volume ($M_b/S, %$)
Figure 6: Cost of Entry
The left panel illustrates an example of a cost of entry function as discussed in Section 2.2. The temporary CDS ban leads to a small decrease in the value of trading as a high-valuation investor. When the scale of entry is already large and due to the convexity of $c(\rho)$, a tiny decrease in $\rho$ results in a large decrease in the cost. As a result, the entry rate does not have to change much in response to a temporary ban. In contrast, with a permanent ban, the value of trading as a high-valuation investor decreases considerably. In addition, due to the convexity, as the entry rate $\rho$ decreases, the resulting decrease in the cost of entry becomes less responsive. As a result, the entry rate has to decrease greatly in response to a permanent ban. The right panel shows the implicit short-run dynamics of the cost of entry $c(\rho(t))$.

Figure 7: CDS Market Liquidity as a Function of CDS Market Efficiency ($\lambda_c$)
The plots illustrate Result 1.2 of Section 3.1. They show the comparative statics of the CDS market liquidity variables with respect to CDS market matching efficiency ($\lambda_c$). The parameter values used to generate the plot are in Table 6.
Figure 8: Bond and CDS Market Liquidity as Functions of Bond Market Efficiency ($\lambda_b$)
The plots illustrate Result 2 of Section 3.1. They show the comparative statics of bond and CDS market liquidity variables with respect to bond market matching efficiency ($\lambda_b$). The parameter values used to generate the plot are in Table 6.

Figure 9: Bond and CDS Market Liquidity as Functions of Funding Liquidity ($O_h$)
The plots illustrate Result 3 of Section 3.1. They show the comparative statics of bond and CDS market liquidity variables with respect to the opportunity cost of entering, $O_h$. A decrease in $O_h$ is an exogenous increase in funding liquidity. The parameter values used to generate the plot are in Table 6.
Figure 10: The CDS-Bond Basis
The plots illustrate Corollary 1 of Section 3.1. They show the comparative statics of the CDS-bond basis, \( p_c - \left( \frac{\delta}{p_b} - r \right) \), with respect to CDS matching efficiency \( \lambda_c \) (left), bond matching efficiency \( \lambda_b \) (middle), and the opportunity cost of entering \( O_h \) (right). The parameter values used to generate the plot are in Table 6.

Figure 11: The Effects of Covered and Naked CDS on Bond Market Liquidity.
The plots compare the marginal effects of allowing naked (in solid bold) and covered CDS purchases (in thin dashed) on bond market liquidity. They also show the marginal effect of allowing both types of purchases (in thin solid). See Section 3.2 for the relevant discussion. The parameter values used to generate the plot are in Table 6.

Figure 12: The Effect of Covered CDS when Entry Is Fixed
The plots show the marginal effect of allowing covered CDS purchases on bond market liquidity when the entry rate of high-valuation investors is fixed. See Section 3.2 for the relevant discussion. Parameter values used to generate the plot are in Table 6. The entry rate is fixed at \( \rho = 0.8 \).
E  Tables

Table 6: Calibration Parameters
This table gives the parameter values chosen for the numerical analysis as described in Section 3.3.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond coupon blow</td>
<td>( \delta_b )</td>
<td>0.01</td>
</tr>
<tr>
<td>Flow value of insurance protection</td>
<td>( \delta_c )</td>
<td>0.01</td>
</tr>
<tr>
<td>Hedging benefit of high types through bonds</td>
<td>( x_b )</td>
<td>0.078</td>
</tr>
<tr>
<td>Hedging benefit of high types through CDS</td>
<td>( x_{ch} )</td>
<td>0.074</td>
</tr>
<tr>
<td>Hedging benefit of low types through CDS</td>
<td>( x_{cl} )</td>
<td>0.079</td>
</tr>
<tr>
<td>Cost of risk bearing</td>
<td>( y )</td>
<td>0.06</td>
</tr>
<tr>
<td>Cost of risk bearing</td>
<td>( y_{1,1} )</td>
<td>0.09</td>
</tr>
<tr>
<td>Exogenous flow of high types</td>
<td>( F_h )</td>
<td>0.854</td>
</tr>
<tr>
<td>Exogenous flow of low types</td>
<td>( F_l )</td>
<td>0.035</td>
</tr>
<tr>
<td>Switching intensity of low types</td>
<td>( \gamma_u )</td>
<td>0.1</td>
</tr>
<tr>
<td>Switching intensity of high types</td>
<td>( \gamma_d )</td>
<td>0.5</td>
</tr>
<tr>
<td>Matching efficiency in the bond market</td>
<td>( \lambda_b )</td>
<td>550</td>
</tr>
<tr>
<td>Matching efficiency in the CDS market</td>
<td>( \lambda_c )</td>
<td>120</td>
</tr>
<tr>
<td>Supply of bonds</td>
<td>( s )</td>
<td>1</td>
</tr>
<tr>
<td>Bargaining power of buyers and sellers</td>
<td>( \phi )</td>
<td>0.5</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>( r )</td>
<td>0.04</td>
</tr>
<tr>
<td>Value of the outside option</td>
<td>( O_h )</td>
<td>0.0361</td>
</tr>
</tbody>
</table>

Table 7: Calibration Results: Volume of Trade and Search Times
This table numerically shows the marginal effects of naked and covered CDS positions on volume of trade and search times as described in Section 3.3.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>No CDS</th>
<th>Naked</th>
<th>Covered</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond volume (% of debt)</td>
<td>( M_b/S )</td>
<td>49.1</td>
<td>49.2</td>
<td>49.1</td>
<td>49.2</td>
</tr>
<tr>
<td>CDS volume (% of notional)</td>
<td>( M_c/(\mu_{0,-1} + \mu_{a1,-1}) )</td>
<td>56.0</td>
<td>2760.</td>
<td>80.9</td>
<td></td>
</tr>
<tr>
<td>CDS notional (% of debt)</td>
<td>( (\mu_{0,-1} + \mu_{a1,-1})/S )</td>
<td>34.5</td>
<td>0.319</td>
<td>34.8</td>
<td></td>
</tr>
<tr>
<td>Days to buy a bond</td>
<td>( 250/(\lambda_b\mu_{bs}) )</td>
<td>25.9</td>
<td>29.4</td>
<td>25.5</td>
<td>29.</td>
</tr>
<tr>
<td>Days to sell a bond</td>
<td>( 250/(\lambda_b\mu_{bb}) )</td>
<td>8.94</td>
<td>7.86</td>
<td>9.08</td>
<td>7.96</td>
</tr>
<tr>
<td>Days to buy CDS</td>
<td>( 250/(\lambda_c\mu_{cs}) )</td>
<td>36.0</td>
<td>142.</td>
<td>50.7</td>
<td></td>
</tr>
<tr>
<td>Days to sell CDS</td>
<td>( 250/(\lambda_c\mu_{cb}) )</td>
<td>74.8</td>
<td>41.6</td>
<td>36.5</td>
<td></td>
</tr>
<tr>
<td>Entry rate</td>
<td>( \rho )</td>
<td>0.605</td>
<td>0.799</td>
<td>0.606</td>
<td>0.8</td>
</tr>
<tr>
<td>Naked CDS (% of total)</td>
<td>( \mu_{0,-1}/(\mu_{0,-1} + \mu_{a1,-1}) )</td>
<td>99.2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 8: Calibration Results: Prices and Bid-Ask Spreads
This table numerically shows the marginal effects of naked and covered positions on prices and bid-ask spreads. See the discussion in Section 3.3.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>No CDS</th>
<th>Naked</th>
<th>Covered</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond bid-ask (%)</td>
<td>$\omega_b/p_b$</td>
<td>2.51</td>
<td>2.14</td>
<td>2.33</td>
<td>1.98</td>
</tr>
<tr>
<td>Expected bond return (%)</td>
<td>$\delta_b/p_b$</td>
<td>6.23</td>
<td>6.00</td>
<td>6.15</td>
<td>5.93</td>
</tr>
<tr>
<td>CDS bid-ask (%)</td>
<td>$\omega_c/p_c$</td>
<td>13.03</td>
<td>3.32</td>
<td>13.01</td>
<td></td>
</tr>
<tr>
<td>CDS price (%)</td>
<td>$p_c$</td>
<td>1.98</td>
<td>3.01</td>
<td>2.00</td>
<td></td>
</tr>
</tbody>
</table>

F Institutional Details and the Bans
This section describes sovereign bond and CDS markets and the European regulations that banned naked CDS purchases. For more details, see Sambalaibat (2014).

F.1 Sovereign Bond Market
A majority of government bonds are traded OTC. Traders shop for sovereign bonds using phone calls, emails, electronic messaging systems and Bloomberg quotes. Locating a particular bond issue can be at times impossible.

Bond markets of few governments are organized as electronic markets. The U.S. Treasury market is the primary example.\(^{36}\) A non-trivial portion of Italian government bonds trades on an inter-dealer trading platform, called the MTS. The MTS functions similarly to an electronic limit order market and is not accessible to individual investors. It is not clear exactly what proportion of all Italian government bonds is traded on the MTS versus OTC. Despite the fact that the MTS is organized like an equity market and is one of the largest and most liquid government bond markets, trade is fragmented across heterogeneous bonds and liquidity per bond is low.\(^{37}\) According to Pelizzon, Subrahmanyam, Tomio, and Uno (2013), daily trading volume and the number of trades per bond on the MTS are comparable to the U.S. municipal and corporate bond markets.

F.2 The CDS Market
As discussed before, CDSs are OTC derivative contracts that resemble insurance protection against a default or a similar event (referred to as a “credit event”) on bonds of a firm or a government (the “reference entity”). A buyer of CDS protection pays a periodic fee (equivalently, the CDS price, premium, or spread) until either the contract matures or a credit event occurs. In return, the seller pays the buyer the protection amount that was purchased (called “notional”) in the event of default (or a similar event) on any one of the bonds of the reference entity that is covered by the CDS contract. CDS contracts are therefore written at the level of firms and governments, not at the level of individual bond issues.\(^{38}\)

CDS contracts specify the reference entity, the contract maturity, the notional amount, the set of bonds of the reference entity that the contract covers, and the default events that constitute a


\(^{37}\)See Cheung, Rindi, and De Jong (2005), Dufour and Skinner (2004), and Pelizzon, Subrahmanyam, Tomio, and Uno (2013) for more information on the MTS trading platforms.

\(^{38}\)For example, suppose you are a holder of bond “A” of the Greek government and Greece defaults on another bond “B.” If both bonds are covered by the contract, you will be still be paid out even if your bond “A” has not been defaulted on.
credit event. The standard notional amounts are in the range of $10–20 million. Prices of CDS contracts are paid quarterly and are quoted as annualized percentages of the contract notional.

The governing body for the CDS market, the International Swaps and Derivatives Association (ISDA), determines whether a credit event has occurred. The standard credit events for sovereign CDS are Failure to Pay and Debt Restructuring. Protection buyers get paid the difference between the notional and the recovery value that is determined through a special post-credit-event auction. For example, if an investor bought a CDS contract with a notional of $10 million and the recovery rate is 25%, she receives $7.5 million in cash. The ISDA finalizes the actual list of eligible bonds that can be submitted into the auction and oversees the auction. At the end of the auction, all bonds submitted into the auction are bought and sold at the same final bond price, and this final price is the price or the recovery rate that settles all CDS contracts on that reference entity. The recovery value is effectively the price of the defaulted bonds. Although cash settlements have become standard now, CDS buyers also have the option of physically settling their contracts by selling their bonds during the auction.

F.3 The Permanent CDS Ban

Throughout 2011, market participants faced uncertainty over whether the EU would adopt measures to ban naked CDS. The uncertainty was finally resolved on October 18, 2011 when, after months of negotiations, the European Parliament and the EU states passed a law to permanently ban naked CDS. The legislation applied to all CDS transactions referencing governments of the EU regardless of the geographic location of the transaction or the legal jurisdiction of the financial institution involved in the transaction.

The final draft of the law was published March 2012 (Regulation EU No 236/2012). Although the legislation was to be in effect beginning November 1, 2012, the March 2012 regulation stated that traders who enter new contracts after March 2012 would have to unwind them by November 2012. Contracts entered before March 2012 could remain in place even beyond November 2012.

Figure 13 compares the total CDS purchased referencing governments affected by the ban (that is, EU governments) versus countries not affected by the ban (non-EU governments). We see that the total amount of CDS purchased on EU sovereigns began to decrease dramatically starting around the time that the law was passed and continued to decline through the end of the sample period. This decrease did not occur for countries not affected by the ban. Thus, anticipating the difficulty of renewing contracts beyond March 2012, traders started to decrease their activity already in the fall of 2011.

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39 This is comparable to the most common transaction sizes of 5, 10, 25 million euros in, for example, the MTS Global Market (see Cheung, Rindi, and De Jong (2005)).

40 For example, if the price of a CDS contract with $10 million notional is 200 basis points, the protection buyer pays $0.2 million annually in quarterly installments of $0.05 million. The price of a CDS contract can be thought of as, in its simplest form, the probability of default times one minus the recovery rate. For example, if a one year CDS contract is trading at 200 basis points, and the recovery rate was zero, then the implied probability of default is 2%.

41 Credit events are decided by the “determination committee” of the ISDA, which consists of 10 big dealer banks (e.g Bank of America, Barclays, BNP Paribas, Citibank, Credit Suisse, Deutsche Bank, Goldman Sachs, JPMorgan, Morgan Stanley, and UBS) and five buy side firms that tend to be hedge funds.

42 For corporate CDS, bankruptcy is an additional standard credit event. There are three types of restructuring that differ by how restrictively they limit the set of eligible bonds: Modified Restructuring (MR), Modified Modified Restructuring (MMR), and Complete (or “old”, “full”) Restructuring (CR). MR is the most restrictive, limiting to bonds with a remaining maturity of at most 30 months after a credit event is declared, then MMR with 60-month maturity, and CR is the least restrictive with the standard 30-year maturity limit on bonds. CDS on North American reference entities usually feature MR (except CDS on high credit risk firms tend to completely exclude any debt restructuring as a credit event), while CDS on European firms feature the less restrictive MMR. Debt restructuring on sovereign CDS, on the other hand, most commonly specify CR.
The regulation specified what would be considered a “covered” CDS purchase and, consequently, any purchase that did not fit the definition of covered CDS purchase was considered a “naked” purchase. A CDS purchase was considered covered if it was hedging a portfolio of assets that was correlated with government bonds of the reference entity.\footnote{Market participants were generally confused about how to actually interpret and satisfy the restrictions of the regulation.} In particular, the value of the portfolio had to have a historical correlation of at least 70% with the government bond price over a period of at least 12 months prior to the CDS purchase. The underlying portfolio could consist of, for example, long positions in private entities within the reference entity country or even long positions through CDS itself. The correlation requirement would be automatically satisfied if the underlying position being hedged consisted of government bonds (at federal and local levels of the government), the liability of state enterprises, and the liability of enterprises guaranteed by the sovereign. The legislation exempted market making activities.

After the purchase, traders did not have to maintain the correlation throughout the CDS contract to allow for prices of the underlying assets to vary. But the size of the underlying positions had to remain “proportional” to the amount of CDS purchased. In other words, a trader could not buy bonds with the intent of selling them back once she purchases CDS. The regulation was enforced by putting the responsibility on institutions to keep track of their positions. Upon request, institutions were supposed to be able to prove that their CDS purchases for the purpose of hedging.

Figure 13 shows the time series of the total amount of CDS purchased around the ban. Between the EU’s vote on the ban in October 2011 and the end of my sample period in June 2013, the total amount of CDS purchased referencing governments of the EU declined by one third, while a similar decline did not occur for countries not affected by the ban (in dashed line). Today, trading in European sovereign single-name contracts has essentially dried up according to ISDA (2014). These observations suggest that naked CDS positions played an important role in the CDS market and possibly constituted a large proportion of the total CDS outstanding.

Figure 13: Permanent Naked CDS Ban and CDS Purchased, Jan 2011–Aug 2012
The solid line plots the total CDS purchased (CDS net notional, $bln) referencing governments subject to the EU ban (that is, EU governments). The dashed line plots the total CDS purchased referencing governments not affected by the ban (that is, non-EU governments). The vertical line is drawn on October 18, 2011 when the EU passed the naked CDS ban legislation. See Section F for more details.

Figure 14 plots the cross country average of the bond bid-ask spread for two groups of countries: affected and unaffected. After the regulation was introduced, the countries affected by the ban

\[\text{Not subject to the ban (R)}\]
\[\text{Subject to the ban (L)}\]
observed a widening of the bond bid-ask spread compared to countries not affected by the ban. This suggests that the ban had a detrimental effect on liquidity of the underlying bonds.

Figure 14: Permanent Naked CDS Ban and Bond Bid-Ask Spreads, Jan 2011–Aug 2012
The solid line plots the cross-country average bond bid-ask spread (% of the mid price) across the countries subject to the ban (EU countries). The dashed line plots the average bond bid-ask spread across countries that were not affected by the ban (outside the EU). The vertical line is drawn on October 18, 2011 when the EU passed the naked CDS ban legislation. See Section F for more details.

F.4 The Temporary CDS Ban
On Tuesday, May 18, 2010 Germany prohibited naked CDS purchases referencing Eurozone governments. As recent as a month prior to the ban, Germany’s rhetoric had been that there is no need to ban naked CDS trading. The regulation was unexpected by market participants and was implemented within the same day of the media’s first report on it. News about the ban first appeared around 1pm on Tuesday, May 18, 2010, on Reuters. But the official details of the legislation did not emerge until late in the evening around 9:30pm. The regulation was effective from midnight the same day (within two and half hours from the release of the official statement) and was temporary in that it was to be in effect through March 31, 2011. On July 27, 2010 the regulation was made permanent.

The regulation also banned the naked short selling of 10 leading German financial stocks and the naked short selling of Eurozone government bonds that were allowed to be listed on Germany’s domestic stock exchange. The naked bond short-selling restriction, as a result, applied to only a few German and Austrian bonds.

The May 18, 2010 regulation did not specify the territorial scope of the regulation. So it is not clear whether market participants interpreted the regulation to apply to all transactions regardless of the geographic location and institution. However, according to Allen & Overy LLP and ISDA’s conversations with BaFin (Germany’s financial regulatory body), BaFin confirmed that the regulation applied to transactions where at least one of the counterparties is located in Germany. It would not, for example, apply to a transaction between the New York branch and the London branch of Deutsche Bank. At first glance, the temporary ban may seem limited in scope. However, it applied to one of the major players in a market already concentrated among a few dealers: Deutsche Bank (see Siriwardane (2015) on the concentration of the CDS market). Thus, the ban seemed to have affected a nontrivial subset of the CDS market with a nontrivial price impact.

Figure 15 plots the cross country average bond bid-ask spread. The dashed line shows the
average for the EU countries that were not affected by the ban (i.e., naked CDS referencing these countries could still be purchased), while the solid line plots the average for the EU countries affected by the ban (i.e., the Eurozone countries). Two vertical lines are drawn for the week before the ban and the week of the ban. We see that for the countries affected by the ban, there was a large and sudden narrowing of the bond bid-ask spread, while this did not occur for the countries not affected by the ban. Figure 16 shows the time series of CDS net notional around the ban.

Figure 15: Temporary Naked CDS Ban and Bond Bid-Ask Spreads, Mar 2010–Aug 2010
The solid line plots the average bond bid-ask spread (% of the mid price) across Eurozone countries. The dashed line shows the average bond bid-ask across EU countries for which naked CDS could still be purchased. The vertical lines are drawn the weeks before and after the German ban. See Section F for more details.

Figure 16: Temporary Naked CDS Ban and CDS Purchased, Mar 2010–Aug 2010
The solid line plots the total CDS net notional ($billion) across Eurozone countries. The dashed line shows the total for EU countries for which naked CDS could still be purchased. The vertical lines are drawn the weeks before and after the German ban. See Section F for more details.
Table 9: Anecdotal Evidence of How the EU Ban Affected the Bond Market

In February 2013, the European Securities and Market Authority (ESMA) surveyed market participants on the effects of the EU naked CDS ban. Below are some responses to Question 15 in the survey (Have you noticed any effect of the prohibition on entering into an uncovered sovereign CDS transaction on the price and on the volatility of the sovereign debt instruments?). For more information on the survey and the responses received from private institutions and industry associations, see http://www.esma.europa.eu/consultation/Call-evidence-evaluation-Regulation-short-selling-and-certain-aspects-credit-default-sw#responses and ESMA (2013).

The German Banking Industry Committee:

“The market has become less liquid; the bid-offer spread has widened. Volatility is unchanged, but has tended to shift to the spot/cash markets.”

The Association for Financial Markets in Europe (AFME) and the International Swaps and Derivatives Association (ISDA):

“Market participants have already observed that seemingly due to the SSR Regulation (restrictions it imposed on sovereign debt and sovereign CDS markets), Asian participation in the European bond market fell by around 50% immediately after the introduction of the SSR, thus demonstrating neatly one adverse impact of the SSR in general in driving investors away.”

“Some buy side market participants have already remarked that even though there is still liquidity in sovereign debt, it is more difficult to source this liquidity.”

Alternative Investment Management Association (AIMA) and Managed Funds Association (MFA):

“Some of our members have reported that they have stopped trading European sovereign CDS and bonds, given the regulatory and reputational risks.”

“Restrictions on CDS positions over the medium term will generally make it more difficult for sovereign issuers to borrow through long-dated securities, leading to a shortening of the average maturity profile of sovereign issuance as investors seek to limit their risk exposure, thereby increasing the vulnerability of sovereigns to short term liquidity and funding crises. This sentiment is reflected in the responses to AIMA and MFA’s poll of their members.”

“At worst, the ban could ultimately undermine liquidity in the underlying sovereign debt markets, undermining the ability of sovereigns to raise finance through debt issuance.”

Deutsche Bank

“We observed anecdotally that as investors began to understand the details of the regulation, cash volumes reduced with a resultant increase in volatility, although this was not significant.”
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