Incentive Effects and Pricing of Contingent Capital\textsuperscript{1}

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Abstract

Contingent Capital bonds—known as contingent convertibles (CoCos)—are bonds that automatically write-down or convert to equity when the financial health of the issuer (typically a bank) deteriorates to a pre-defined trigger. Using a dynamic model of capital structure we show that if conversion terms are dilutive for existing shareholders, banks will have incentive to reduce likelihood of triggering by pursuing lower leverage, leading to less defaults and lower borrowing costs. Conversely, if at conversion, bond principal is written down without diluting shareholders, then banks will have perverse incentives to pursue higher leverage and capital destructive policies (money burning).

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Abstract

Contingent Capital bonds—known as contingent convertibles (CoCos)—are bonds that automatically write-down or convert to equity when the financial health of the issuer (typically a bank) deteriorates to a pre-defined trigger. Using a dynamic model of capital structure we show that if conversion terms are dilutive for existing shareholders, banks will have incentive to reduce likelihood of triggering by pursuing lower leverage, leading to less defaults and lower borrowing costs. Conversely, if at conversion, bond principal is written down without diluting shareholders, then banks will have perverse incentives to pursue higher leverage and capital destructive policies (money burning).
1 Introduction

This paper proposes a model of dynamic capital structure to quantify and value the ex-ante incentive effects of Contingent Capital bonds – also sometimes known as contingent convertibles (or CoCos). Cocos are bonds that are automatically written-down or converted to equity when the issuer’s financial health deteriorates to a pre-defined threshold or trigger. The most natural issuers of contingent capital are often assumed to be banks, not least because Basel III requires that capital-qualifying debt be loss-absorbing. But the model insights and quantitative results that we discuss in this paper have general implications for capital structure theory.

There are two primary advantages of CoCos, we argue. The first, which is well-known, is that CoCos can automatically recapitalize a bank when it is overleveraged and likely in distress, but still has significant enterprise value. When conversion is triggered, leverage is reduced, no new external funds are raised from capital markets, and no government funds are needed for bailouts. For this reason, CoCos are currently viewed as a potentially valuable tool for helping bank regulators address the “too big to fail” problem (see the discussion in Flannery, 2009a, b).4

The second advantage – the one that is the focus of this paper, but has been largely ignored by researchers and regulators – is that CoCos can, if properly designed, create a strong incentives for banks to pursue conservative capital structures. In order to understand these incentives, we need to pay particular attention to one important design feature of the contingent capital, namely, the conversion terms. These terms determine whether at the trigger event, the losses are absorbed by the bank shareholders or by the holders of contingent capital. If the conversion terms are dilutive for the pre-existing shareholders (meaning that

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4See also the discussion by Squam Lake Working Group on Financial Regulation (2009) and Duffie (2009).
one dollar of bond’s par value converts to *more* than one dollar of equity), then banks have an incentive to maintain a precautionary equity buffer (i.e., lower leverage). In response to negative capital shocks, banks would face greater risk of future incremental dilution from forced conversion of the CoCo, and thus would have strong incentives to preempt this, for example, by raising new equity.5

We show that (dilutive) CoCos can mitigate the ex-post conflict of interest between shareholders and bondholders known as the “debt overhang problem.”6 As is well-known, a bank financed with straight debt and managed on behalf of shareholders has no incentive to issue equity during financial distress because the cost of dilution exceeds the benefit of lower financial distress with the most of the benefits going to bondholders. Among other phenomena, this logic can explain why banks that are overlevered and vulnerable to distress are often reluctant to issue equity, and may help explain why bank managements often view equity as a relatively expensive form of external financing.7 Debt overhang also generates an incentive to pursue high-risk but low-return investments (“risk shifting”), amplifying the deterioration of asset quality during economic downturns. We argue that CoCos, if properly designed, can create incentives to mitigate the above consequences of debt overhang.

We quantify the competing incentive effects of contingent capital and debt overhang using a dynamic model of capital structure that trades off the cost of issuing equity (and foregoing the equity benefit of debt overhang) against the cost of accepting the punitive terms dictated by the CoCo contract. We are particularly interested answering such questions as “how dilutive do the conversion terms need to be in order to materially influence management

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5The incentive consequences of contingent capital have also been commented on by Strongin, Hindlian, and Lawson (2009), Pitt, Hindlian, Lawson, and Himmelberg (2011), Berg and Kaserery (2011), and Calomiris and Henning (2011).


7The reluctance to issue equity may also be motivated by adverse selection premium as suggested by Akerlof (1970) and Leland and Pyle (1977).
incentives?” and “how large does the CoCo tranche need to be?” The model allows us to quantify (and price) the incentive effects of tranche size and conversion ratio by quantifying expected future changes in equity issuance, leverage, and default risk alongside the prices of common equity, contingent capital, and senior debt.

For our baseline case – where we add market frictions such costs of issuing equity and “jump risk”– even a relatively small tranche of contingent capital bonds where conversion terms are dilutive to shareholders (as described above) can go a long way toward neutralizing the negative incentive effects of debt overhang. We show that managers acting on behalf of shareholders would issue new equity preemptively to move away from the trigger boundary to reduce the equity dilution costs associated with CoCo conversion. As such, a tranche of contingent capital in the capital structure effectively pre-commits management to a more conservative capital structure. The material reductions in leverage, default risk, and credit spreads can be created with tranche sizes that are a relatively small fraction of debt outstanding. For example, we show that a CoCo tranche equal to 5% of assets, a conversion ratio of 8% above par, and a trigger set at 6% capital results in a reduction of credit spreads by about one quarter and lowers default probability by about one third when compared to a similar CoCo with a conversion ratio set at par.

Since such bonds can lower expected default losses and thus reduce the cost of both CoCo debt and senior debt at issuance, it is natural to wonder why firms do not voluntarily choose to issue CoCos. Part of the explanation may again be traced to ex-post conflicts of interest: if (dilutive) CoCos are issued mid-stream, the value of committing to lower future leverage ratios flows mostly to the pre-existing (senior) debt rather than to the equity. Thus, just as the logic of debt-overhang explains the ex-ante value of adding CoCos to the capital structure, it also explains why banks would not voluntarily choose to issue them in the absence of regulatory requirements or incentives.
We also show that real-world market frictions can weaken the effect of incentive for banks to issue equity. First, we quantify the extent to which beneficial incentive effects are obviously diminished by costs of issuing equity (such costs may arise from explicit transaction costs or from managements view that its equity is undervalued). Second, we quantify the extent to which incentives are diminished by “jump-to-default” risk (representing, for example, bank runs). Such jumps dampen incentives because they reduce the extent to which management can reduce conversion risk by holding excess capital. Third, we show that incentives are dampened by “trigger uncertainty” (or “trigger ambiguity”). In practice we see trigger uncertainty arising in a number of ways, but two stand out. On the one hand, triggers based on regulatory capital are often thought to be vulnerable to “regulatory failure” resulting in delayed triggering and conversion. Consistent with intuition, our results suggest that the risk of such delays weakens incentives. This underscores the value of enhanced disclosure and transparency as mechanisms for increasing political scrutiny of regulators. On the other hand, triggers based on market values of equity are often thought to be vulnerable in the other direction – to “market failure” resulting in premature triggering and conversion (including market failures due to strategic behavior of investors large enough to manipulate equity prices). Again, consistent with intuition, our results show that the risk of early triggering weakens incentives. While the details of trigger design go beyond the scope of this paper, our results may suggest the advantage of “hybrid” or dual trigger designs that combine both regulatory and market-based features.\(^8\) For example, to guard against regulatory failure, a regulatory trigger might be augmented by a requirement that regulators initiate an investigation once market equity falls below a trigger value. Conversely, to guard against market failure, a market-based trigger might be similarly augmented by a regulatory review.

We finally argue that failing to provide regulatory incentives to re-issue CoCo bonds –

\(^8\)See discussion in McDonald (2010).
following conversion or maturity – further dampens the incentive effects of CoCos. Once the initial tranche of CoCos has been triggered, shareholder incentives are back in a world of debt-overhang. And as argued above, shareholder will avoid issuing dilutive CoCo bonds “midstream” because the subsequent benefits of conservative leverage flow disproportionately to existing bondholders. Since shareholders can anticipate this future state of the world, incentives even pre-conversion are dampened.

This disincentive to re-issue raises an important practical question for bank regulators: Once a bank’s CoCo bonds have been triggered (conservative leverage notwithstanding), what can be done to enforce or encourage re-issuance? One idea might be to prohibit capital payouts as well as any issuance of long-term debt until a minimum level of CoCo bonds has been re-issued. While an in-depth treatment of regulatory design is beyond the scope of this paper, our model results illustrate the importance of such considerations for incentives and pricing. For example, we show that incentives to issue equity are increased if dividend payout restrictions are imposed following conversion of the CoCos, because such restrictions reduce the equity option value of debt overhang. Thus, to extract the maximum incentive benefits of CoCo bonds, bank regulation would theoretically need to mandate not just their initial issuance, but also their subsequent re-issuance, post-conversion. One approach might be for regulators to prohibit dividends, share repurchases, and new debt issues until new CoCos have been issued up to the minimum required tranche size.

It is important to emphasize that desirable incentive effects of CoCos are reversed for the case when conversion terms impose losses on CoCo holders rather than on bank shareholders. If conversion ratio is less than one-to-one – that is, the par value of the CoCo is simply written down, such contracts are therefore accretive (rather than dilutive) to shareholders. Our model shows that accretive conversion terms create perverse incentives for increased leverage, or worse, “money burning.” The logic is straightforward: When the bank is close
to the trigger, it can “burn capital”, accelerate breaching the trigger, write-down the bond’s principal, and thus capture a windfall gain for shareholders at the expense of the CoCo investors (see a simple numerical example in appendix 2 that illustrates these incentives). Such bonds thus make the debt-overhang problem even worse than it is for straight debt (see Berg and Kaserery, 2011), and thus magnify the usual “debt overhang” incentives (and hence increase risk shifting, underinvestment, and social costs of default). This intuition does not appear to be widely appreciated either by academics or by policymakers. On the contrary, the vast majority of CoCos issued to date (mostly by European banks) are accretive rather than dilutive to shareholders (for more details, see appendix 1), evidently without objection from regulators. This lack of appreciation for the value-destroying incentives of accretive conversion terms has potentially serious consequences for the future behavior of banks in the next financial crisis.

The model in this paper helps to formalize the logic of money-burning incentives and also to calibrate the magnitude of value destruction in the event of financial distress. Our numerical results show, for example, that if a bank has issued a tranche of CoCo bonds equal to 5% of total assets, and with principal write-downs of 75% upon triggering, then as the bond approaches maturity, shareholder value is maximized by “burning” as much as 3% of assets to accelerate conversion. For the $13 trillion of assets held by U.S. banks, in other words, this could imply the additional value destruction in the event of a future crisis of nearly $400 billion.

The incentives created by accretive (“write-down”) conversion terms are obviously objectionable since they give banks the incentive to destroy additional value as they spiral into

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9 Our notion of “money burning” is driven by adverse incentives rather than adverse selection, and is thus distinct from “costly signaling” intuition described by Spence (1973). The “money burning” in our setting resembles the net dividend payout in a tax-disfavored form discussed in Bernheim and Redding (2001) with a significant difference that we do not assume asymmetric information.
financial distress. And of course, from a policy maker’s perspective, it is exactly during the crisis that is important to have mechanisms in place that create rather than destroy incentives to preserve total asset value.

To the best of our knowledge, this paper is the first to quantify and price the dynamic capital structure incentives created by CoCo bonds – and their terms – in a structural pricing model. In addition to research that has noted the possible incentive effects of contingent capital (e.g., Strongin, Hindlian, and Lawson (2009), Pitt, Hindlian, Lawson, and Himmelberg (2011), Berg and Kaserery (2011), and Calomiris and Henning (2011)), our paper relates to existing research that develops pricing models of contingent capital. Albul, Jaffee, and Tchistyi (2010) derive an analytical solution for the market value of CoCo debt in an infinite maturity setting with a static capital structure. Pennacchi (2010) models both the bank’s risky assets and its deposits and solves for the market price for contingent capital debt and the fair deposit insurance premium. Pennacchi, Vermaelen and Wolff (2011) propose an alternative form of contingent capital in which the shareholders have an option to buy shares back at par value from the CoCo holders after conversion. This option allows shareholders “undo” a potential wealth transfer to CoCo holders caused by conversion terms or market manipulation. McDonald (2010) introduces and prices a contingent capital claim with a dual trigger that depends on both the bank’s own stock price falls as well as a broad financial index of financial distress. Finally, Sundaresan and Wang (2011) show that conversion triggers based on market value of bank’s stock can result in multiple equilibrium prices (or no equilibrium price) for contingent capital. In contrast to our model, these models all assume that future capital structure is determined by the exogenous process for assets, thus precluding the incentive effects of contingent capital on capital structure choices.

The rest of our paper proceeds as follows. The next section develops the model, section 3 presents numerical results and comparative statics of the model, and section 4 concludes.
Following this, appendix 1 briefly describes the contractual terms of some of the existing CoCo bonds, appendix 2 presents a simple one-period numerical example that illustrates the incentives effects, and appendix 3 provides more details on the dynamic stochastic control problem.

2 Model

2.1 Model description and time line

We consider a dynamic structural model of a bank operating in continuous time. Bank assets are stochastic (with jumps) and cash flows are proportional to assets. Taxes are zero, and dividends are paid as a residual after coupon payments.\textsuperscript{10}

At time 0, the bank issues both the CoCo debt and senior debt in its capital structure. Both CoCo bond and senior debt assume coupons payments, and both have the same maturity, but the CoCo bond is junior. At maturity, par values are paid to their claimants respectively, assuming the CoCo has not been converted in prior periods.

Conversion of the CoCo is triggered any time when the capital ratio drops below a contractually specified threshold (the model is agnostic on whether the trigger is market-based, accounting-based or regulatory; more on this below). The CoCo bond converts for a fixed \textit{market value} of shares. The conversion ratio determines the market value of equity to be owned by CoCo holders (could be below or above par), where the exact number of new shares is calculated after conversion.\textsuperscript{11} After conversion, the bank continues servicing the remaining senior debt until maturity. If asset value falls further, the bank can choose to default on its senior debt anytime before or at maturity.

\textsuperscript{10} Taxes can be easily introduced to the model. However, a non-zero corporate tax rate in this model will have no material impact of the incentives created by CoCos.

\textsuperscript{11} The model can incorporate the conversion to a fixed number of shares. McDonald (2010) discusses the advantages and disadvantages of these two cases.
The bank can issue equity at any time prior to breaching the trigger, thereby raising its capital ratio and moving away from the trigger. The bank can also choose to “burn money” (waste assets or capital), which allows for the possibility that the bank may be willing to “pay” to reduce its capital ratio and thus force conversion. Equity issuance, money burning and default are endogenous and chosen to maximize the existing shareholders’ wealth, i.e., current share price.\textsuperscript{12}

We augment the model described above with a number of frictions designed to accommodate practical design considerations arising from real-world concerns:

- **Transaction costs for equity issuance.** When the bank issues equity, it incurs transaction costs that have fixed and variable components. Such frictions are consistent with the lumpiness and relative infrequency of equity issues observed in the data\textsuperscript{13}, and their inclusion allows us to calibrate the extent to which dynamic capital structure incentives are correspondingly reduced.

- **Negative “jumps” in the diffusion process for the value of bank assets.** Jumps in the process for asset value are meant to capture the fact that financial systems are periodically hit by large crises. Since jumps can more easily swamp precautionary capital buffers held to reduce the risk of undesired (dilutive) CoCo conversion, their inclusion here allows us to calibrate their practical significance.

- **Uncertainty in the trigger mechanism.** Capital structure incentives of CoCos may be further reduced by the expectation of two possible types of trigger failure. The first is the “type I” error that the CoCo fails to trigger despite capital ratios having breached the minimum threshold (this risk might arise due to, say, poor disclosure or regulatory

\textsuperscript{12}For simplicity, we assume that the bank cannot sell assets and use the proceeds to repurchase debt or shares.

\textsuperscript{13}Such frictions are commonly thought to reflect information frictions; see Loughran and Ritter (1995).
failure). The second is the “type II” error that the CoCo accidentally triggers even though the bank’s capital is still above contractually specified threshold (this risk might arise due to, say, a noisy market trigger or overly-aggressive regulatory intervention). We calibrate the incentive and pricing effects of both types of trigger failure.

Figure 1 illustrates the time line of the model by showing three representative sample paths of amounts outstanding for assets, total debt and CoCos. Along sample path 1 (the bold line), the bank issues equity at time $t_1$ and converts at time $t_2$ when bank assets drift to the trigger boundary. After time $t_2$, asset value remains above the default boundary and the bank pays off its senior debt at maturity. On sample path 2, asset values drift below the default boundary and the bank defaults. On sample path 3, a medium-sized negative jump arrives which drops asset value through the trigger boundary, thus converting the CoCo. Finally, on sample path 4, a negative jump arrives which is so large that it wipes out both the equity value and the CoCo bonds, causing immediate default without triggering conversion.

2.2 Value of the bank capital

We assume that when there is no injection of capital (or money burning) and no arrival of jumps, the value of the bank’s assets follows a log-normal stochastic process:

$$\frac{dV}{V} = (r - \alpha)dt + \sigma dW_V,$$

where $r$ is a short-term risk-free rate which is assumed to be constant; $\alpha$ is the payout rate; $W_V$ is a Weiner process under the risk-neutral measure; and $\sigma$ is the instantaneous volatility coefficient. Given payout rate of $\alpha$, bank’s assets generate continuous cash flows of $\alpha V$. Equityholders can choose to increase (decrease) the size of the bank’s capital by issuing equity (burning money) of any size. If the bank decides to issue equity (burn money) and
raise (reduce) capital by $\Delta V$, the value of the bank’s assets at time $t+$ will be:

$$V_{t+} = V_t + \Delta V,$$

(2)

where $\Delta V > 0$ is the size of newly raised equity (or money burned, if $\Delta V < 0$). We assume that when the bank issues equity and raises capital, it incurs transaction costs $TC$, which have both a fixed component (proportional to the level of its current assets) and a variable component (proportional to the size of newly raised assets $\Delta V > 0$). As such, if the bank issues equity and raises the size of its assets by $\Delta V$, total transaction costs $TC = e_1 V_t + e_2 \Delta V$, and $e_1$ and $e_2$ are positive constants. These transaction costs are paid by the equityholders of the bank. For simplicity, we assume that there are no transaction costs if the bank burns money.

We also assume that the bank’s asset value can drop discontinuously due to the random arrival of negative “jumps.” Such jumps allow for the possibility of shocks that cause asset value falls far enough (and fast enough) to cause CoCo conversion or even default. Such jumps represent a real-world “friction” in the sense that they preclude the possibility of the firm issuing equity fast enough to prevent conversion (or default).

Formally, we let the independent and uniform random variable $Y \in [0, 1]$ describe the magnitude of the shock as a percentage of assets. Arrival times are independent and follow a Poisson process (Merton (1990)). Specifically, the probability that a jump arrives during time interval $\Delta t$ is $\lambda \Delta t$, where $\lambda$ is a risk-neutral arrival intensity describing the expected number of jumps per year. The expected change in assets due to jumps is $-\lambda k$, where $k = \mathbb{E}^Q(Y)$ and $\mathbb{E}^Q$ is the expectation under the risk neutral measure $Q$. The value of bank’s assets is therefore described by:

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14Pennacchi (2010) and Andersen and Buffum (2002) also assume jumps in asset value process.
\[
\frac{dV}{V} = (r - \alpha - \lambda k)dt + \sigma dW_V + dq,
\]  

where \( dq = (Y - 1)dN_t \) describes fluctuations in bank assets due to jumps, where \( N_t \) is a Poisson process. Figure 2 provides an illustration of the two types of sample paths: the ordinary diffusion process and the jump-diffusion process with asymmetric jumps. Note that both processes have the same long-run drift.

### 2.3 Senior debt and contingent capital

The bank is assumed to have both senior debt and contingent capital bond in its capital structure. Senior debt and contingent capital require continuous coupon payments at the rate of \( f \) and \( c \), respectively, and principal payments of \( F \) and \( C \) at maturity. Given a payout rate of \( \alpha \) from bank’s assets, the bank generates continuous dividends net of interest payments of \( \alpha V - f - c \).

In the baseline model, we assume that there is no trigger uncertainty or trigger failure. If at any time before maturity the bank’s capital ratio, \( \frac{V - F - C}{V} \), falls below the predetermined threshold \( \theta \) (conversion trigger), the CoCo bond automatically converts to a fixed dollar amount of shares. Particularly, the terms of conversion pre-specify that each $1 of CoCo bond converts to $\pi$ of market value of equity, where \( \pi \) is a conversion ratio.\(^{16}\) If \( \pi > 1 \), conversion dilutes existing equityholders because it transfers wealth from equityholders to CoCo investors. At conversion, the existing equityholders transfer shares valued at \( \pi C \) to CoCo holders and the bank’s debt size declines by \( C \) from \( F + C \) to \( F \). Post-conversion, the

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\(^{15}\)When the bank operates with negative dividends, i.e., \( \alpha \cdot V - f - c < 0 \), it corresponds to the case where it continuously injects equity to make interest payment.

\(^{16}\)These two assumptions differ from assumptions in Sundaresan and Wang (2011). In their model, they assume a market-based trigger where both the conversion trigger and conversion price themselves depend on the market prices of the bank’s shares. They show that this inter-dependence may result in multiple equilibria or even in an absence of equilibrium price of the bank’s equity.
bank continues servicing the remaining debt, so dividends are \( \alpha V - f \). At maturity, the bank repays the par value of \( F \).

The bank can endogenously choose to default any time before maturity when its market equity value declines to zero. At default, the equityholders’ value is zero, and debtholders recover the bank’s assets \( V \) minus proportional default costs \((1 - DC)V\), where \( DC \) is a positive constant, \( 0 < DC < 1 \). CoCos are assumed to be junior in bankruptcy, so that CoCo investors are paid only after senior debt recovers 100%.

With jumps in the asset value process, asset value can jump to a level \( V' \) that wipes out existing equity investors, but that still leaves positive residual value available to CoCo investors (i.e., \( \pi C > V' - F > 0 \)). In such cases, the conversion ratio \( \pi \) is modified to allocate the residual equity value to CoCo holders, implying that post-conversion, the bank becomes wholly-owned by CoCo bondholders.

If the size of the realized jump is large enough, the bank assets can fall below both the trigger and default boundaries. In such case the bank will default without triggering the conversion. Given that the CoCo bond is junior, the CoCo holders will have lower priority in recovering value at default, and for most parameter values, will receive zero value.

### 2.4 Base case calibration of parameter values and numerical calculations

The base case parameter values listed in Table 1 are chosen to crudely describe a well-capitalized bank. (Table 2 reports an expanded range of parameter values around the baseline values reported in Table 1.) For the base case, the volatility of bank assets is set at \( \sigma = 5\% \), the risk-free interest rate is \( r = 5\% \), and the asset payout yield \( \alpha \) is 5\%, which means that the risk neutral diffusion drift of assets is 0\% per year. Default costs are assumed to be \( DC = 50\% \) of asset value.
The size of the CoCo tranche, $C$, is assumed to be a relatively small fraction of bank assets. For the base case, it is set to 5% of initial assets, $C = 5\% \times V_0$. The size of senior debt is set to 87% of bank assets, $F = 87\% \times V_0$. Thus, bank’s initial leverage ratio is $\frac{F+C}{V_0} = 92\%$, and its initial capital ratio (book value) is $\frac{V_0-F-C}{V_0} = 8\%$. Both CoCo bond and senior debt mature in five years. The trigger level of capital for the CoCo is set to $\theta = 6\%$ (so that conversion occurs when $V$ satisfies $\frac{V_0-F-C}{V_0} \leq 6\%$), and the conversion ratio is varied from $\pi = 1.0$ (meaning par-to-par conversion) to $\pi = 1.10$ (10% above par).

Transaction costs of raising new equity are assumed to be $e_1 = e_2 = 0.025\%$, reflecting both fixed and proportional components. Butler, Grullon and Weston (2005) document investment banking fees for equity issuance around 5%. If we assume equity issuance is 5% of total capitalization and equity capital is 10% of assets, then total fixed transaction costs are $5\% \times 5\% \times 10\% = 0.025\%$ of asset value.

Finally, we assume that the jump size $Y_t$ is uniformly distributed on $[0, 1]$. This means that conditional on arrival, a jump leads on average to a 50% loss in asset value (parameter $k = -0.5$). The annual probability of jump is $\lambda = 2\%$, representing the economic environment in which financial crisis (bank runs) happen on average every 50 years. Due to jumps, the risk neutral diffusion drift of the value process is adjusted upward by $-\lambda k = 1\%$.

To simplify calculations, we assume the principal value of the senior debt and the CoCo bond paid at maturity are equal to the value of perpetual debt with periodic coupon payment discounted at the risk-free rate, i.e., $F = \frac{F}{r}$ and $C = \frac{C}{r}$, respectively. This implies that, since the senior debt is generally risky, it is sold at a discount relative to its face value, $D < F$, where $D$ is the market value of the senior debt. The CoCo bond can be sold above or below par, i.e., it can be $Z < C$, or $Z > C$, where $Z$ is the market value of the CoCo bond.17 Given

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17 Such an approach simplifies the numerical calculation. Otherwise, we would need extra numerical iterations to find the par coupon rate.
market values of senior debt and CoCo bond, we calculate credit spreads at origination as the difference between bond yield and risk-free rate. We also measure the bank’s average cost of debt by calculating its weighted-average (WA) spread.

We calculate expected default costs by subtracting the total value of debt plus equity from the value of the bank’s underlevered assets: default costs = \( V - E - (D + Z) \). We measure the size of default costs as percentage of total assets. By analyzing the size of these costs as a function of the CoCo contract terms we can gauge its efficiency in reducing expected default costs. We also calculate the expected value of future net equity issuances as well as the expected value of burnt money both measured ex-ante at time zero. Finally, we calculate the default probability and probability of triggering under the real-world probability measure. For these calculations we assume a 3% premium and thus increase diffusion drift from 5% to 8%. Mapping the risk-neutral intensity of the jump process to the real world is more complex. Chen, Liuy and Maz (2007) show that for a recursive utility function, there is a linear relation between the risk-neutral arrival intensity and the real world intensity, which has the following form: \( \lambda = \lambda^{RW} f(\cdot) \), where \( \lambda^{RW} \) is the real-world intensity of jump arrival and \( f(\cdot) \) is a function of the average jump size, volatility of the jump, and the risk aversion coefficient. For our parameterization, we assume that \( f(\cdot) = 10 \), which corresponds to the risk aversion factor of about 3.\(^{18}\) In our comparative statics, we will show the range of CoCo terms that will incentivize the bank to preemptively issue equity.

We assume complete markets for the bank’s assets, so that the CoCo bond, senior debt and equity can be regarded as tradeable financial claims for which the usual pricing conditions must hold. The market values of equity, CoCos, and senior debt are functions of three variables: asset value, \( V \), face value of debt, \( F + C \) (or \( F \) if conversion has already taken place), and time, \( t \leq T \). Equity value solves a stochastic control problems with fixed and free

\(^{18}\)We assume that in the real world probability measure, the distribution of jump size do not change.
boundary conditions, where the control variables are the equity issuance (or money burning) and default. These valuation problems are described in detail in appendix 3.

The numerical algorithm used to compute the values of equity and debt is based on the finite-difference method augmented by policy iteration. Specifically, we approximate the solution to the dynamic programming on a discretized grid of the state space \((V, \{F, F + C\}, t)\). At each node on the grid, the partial derivatives are computed according to Euler’s method. The backward induction procedure starts at the terminal date \(T\), at which the values for senior debt, the CoCo bond and equity are determined by payoff to CoCo holders, holders of the senior debt, and holders of bank equity. The backward recursion using time steps \(\Delta t\) takes into account the bank’s optimal default decision and its optimal strategy to raise capital (or burn assets). For more detail of the numerical algorithm see Titman and Tsyplakov (2005) and Tsyplakov (2007).

### 3 Results

Using a range of plausible parameter values, we use numerical solutions of the model described in the previous section to address the following general questions:

1. How do the contractual terms of CoCos, including tranche size and the conversion ratio, affect management’s propensity to maintain conservative leverage ratios by issuing future equity?

2. To what extent are these incentive effects offset by countervailing frictions, namely issue costs for new equity, negative jumps in asset value, and uncertainty in the trigger mechanism?

3. How does (predictable) future behavior feed back into the market of pricing of equity, CoCo bonds, and senior debt?
We begin with an analysis of base-case parameter ranges that assume “neutral-to-dilutive” conversion terms. Among other results, this analysis shows how capital structures become more conservative as conversion ratios become more dilutive. In the following section, we calibrate a model for which conversion terms are accretive for shareholders, and thus create incentives to burn money.

3.1 The base case

Figure 3 illustrates the consequence of CoCo incentives on dynamic capital structures by comparing two initial capital structures: a capital structure in which the bank is financed with all senior debt, and a baseline case in which CoCo replaces a portion of senior debt. In both scenarios, total leverage is 92%, but in the latter case, CoCos are 5% of assets and senior debt is 87%. Figure 3 displays numerical model solutions showing how equity issuance, expected default costs, credit spreads, and equity values are affected as the CoCo bond’s conversion ratio is increased (i.e., made more dilutive for existing shareholders). At the origin, one dollar of par converts to one dollar of bank’s equity ($\pi = 1.0$). The comparison of two capital structures at this point shows that adding even “par” CoCos to the capital structure can meaningfully reduce the deadweight cost of default relative to the case when the bank is financed with all senior debt. Expected default costs decline from 6.5% to 3.5%, and weighted-average credit spreads fall from 45bp from 69bp. These results confirm earlier studies that CoCo bonds can reduce overall default risk, even if the incentive affects are not enhanced by aggressive conversion terms.

3.1.1 CoCo structure and incentives to issue equity for the base case

The model predicts that even a small tranche of CoCo bonds with moderately dilutive conversion terms can create strong incentives for the bank to issue equity, de-lever and reduce the probability of triggering. The bank’s incentives to issue equity and recapitalize
are greater when it faces more dilutive conversion terms. Holding the size of CoCo at 5% of bank assets, an increase in the conversion ratio from $\pi = 1.0$ to 1.06 will incentivize the bank to increase its future equity issuance rate from zero to 1%. This rate is calculated as the expected equity issuance rate measured over the entire maturity period. As the conversion ratio is increased further from $\pi = 1.06$ to 1.09 and higher, the expected equity issuance rate increases to a steady state level of 6.0% (See Figure 3). For more dilutive conversion ratios for $\pi = 1.09$ and above, the bank will always avoid drifting through the trigger by preemptively raising just enough equity to keep its leverage above the conversion trigger. Due to the presence of negative jumps, however, the risk of triggering cannot be reduced to zero. The credit spreads shown in Figure 3 reflect the benefits of this “precautionary” equity issuance. The weighted-average credit spread narrows and default costs both gradually decrease as the conversion ratio increases.

A more interesting result is a non-monotonic relation between the CoCo spread and the conversion ratio. The CoCo’s spread narrows as the conversion ratio increases from $\pi = 1.0$ to 1.06. However, the spread widens as the conversion ratio increases further from 1.06 to 1.09, and slightly decreases for $\pi > 1.09$. The intuition for this result is the following. Over the range of $1.0 < \pi < 1.06$, the CoCo spread declines because an increase in the conversion ratio will increase the expected “windfall” to the holders of the CoCo debt. At the same time, the probability of conversion (shown in the Figure 3) is falling, but the relative impact is small because the expected rate of equity issuance is still relatively low (below 1%). As the conversion ratio increases from 1.06 to 1.09, however, the bank’s expected equity issuance increases by an order of six (from 1% to 6%). This significantly reduces the probability of conversion, thus resulting in a lower windfall for CoCo holders and a higher spread. At conversion ratios of 1.09 and above, the expected rate of equity issuance rate plateaus at around 6% and hence so does the spread.
3.1.2 The incremental effect of incentives to issue equity

In order to quantify the pricing effect of incentives we also consider the scenario when the bank cannot issue equity and compare the model-generated values for this scenario with the values of the base case. If the option to issue equity is ignored then the weighted-average credit spread, the spread of the senior debt and the default costs will be overestimated. As shown in Figure 4, when the conversion ratio is 8% above par ($\pi = 1.08$) or higher, the pre-commitment to lower future leverage will reduce the bank’s weighted-average credit spreads, default costs and default probability by about one-half when compared to the scenario when the bank cannot recapitalize.

These quantitative predictions confirm the idea that when conversion costs are borne by the equityholders, even a small tranche of CoCo bond can create a strong incentive for a bank to issue equity in response to future capital losses and these incentives can create sizable incremental value by reducing the costs of financial distress.

3.1.3 Will the bank issue CoCo bonds voluntarily?

In this section, we explore whether the bank will be willing to issue a new CoCo bond midstream when it already has the pre-existing senior debt in its capital structure. We will argue that, if the conversion terms dilute shareholders, the value of committing to lower leverage flows mostly to pre-existing senior debt rather than equity.\(^{19}\) As such, the CoCo tranche will create value to holders of existing bonds at the shareholders’ expense.

To assess this potential wealth transfer, we calculate the change in market value of equity and the change in yield of the preexisting senior debt at the instant of CoCo issuance, assuming that the issuance is unexpected. We first consider the bank that initially operates

\(^{19}\) Albul, Jaffee, and Tchistyi, (2010) analyze a similar case when the CoCo bond is issued on top on the pre-existing straight debt. However, they don’t consider the case when the bank can recapitalize.
with the senior debt only (87% of total assets) which is priced without the CoCo tranche. Numerical calculations show that the senior debt has credit spread of 45 bp above the risk-free rate. We then assume that the bank issues a CoCo bond (5% size) with the same maturity as the existing senior debt, and vary its conversion terms. Figure 3 shows that an increase in conversion terms leads to a larger wealth transfer. If the conversion ratios of the newly issued CoCos are 8% above par ($\pi > 1.08$), the yield of the pre-existing senior debt will decline by 12 bp, while the shareholders will lose as much as 2.4% of their stock market value upon issuance. In comparison, with par-to-par conversion terms ($\pi = 1.0$), there will be no value transfer because the bank will not commit to issue equity and de-lever in the future.

This result confirms the intuition that commitment to lower leverage benefits mostly pre-existing holders of senior debt rather than holders of equity. Due to a possibility of wealth transfer, the bank will not voluntarily choose to issue CoCos with dilutive conversion terms when there is already a senior debt in its capital structure.\footnote{These concerns are eliminated should the bank issue CoCo debt at the same time it issues senior debt. If issued simultaneously, the bank’s shareholders will benefit because the CoCo debt will increase the value of the newly issued senior debt resulting in lower overall borrowing costs.} As a policy implication this result suggests that regulators might be needed to mandate the issuance of such CoCos.

3.1.4 Choosing tranche size and trigger ratio

Figure 5 presents the model-generated spreads and other values for three different sizes of CoCo bond where the size is measured as a percentage of initial assets, $\frac{C}{V_0}$. In this test, the size of senior debt and the size of CoCo bond are both varied maintaining the total leverage at 92% (the base case level). Results testify that when the bank has a larger size CoCo tranche, it will have more incentive to preemptively recapitalize to reduce the likelihood of conversion. This is because a larger size CoCo tranche creates larger shareholder dilution at
conversion.

Figure 6 plots the curve representing the set of contracts in the two-dimensional space (where both the CoCo bond size and the conversion ratio are varied) at which expected default costs are calculated to be at the level of 2%. This indifference curve quantifies the tradeoff between the choice of the CoCo size and the magnitude of the “dilution” terms. For example, the CoCo that has size of 5% and converts at 8% above par ($\pi = 1.08$) will be as effective (in generating incentives and reducing default costs) as the CoCo that has size of 8% but converts at par ($\pi = 1.0$).

Figure 7 presents the comparative statics for three cases of trigger ratio (parameter $\theta$) at which the CoCo converts. With the higher trigger (higher value for parameter $\theta$), there will be a higher likelihood of breaching the trigger. If the bank wants to keep its capital ratio above a higher trigger, the bank has to start issuing equity at a higher levels of capital. As such, the expected value of equity issuance rate will be higher and expected default costs will be lower. Figure 7 confirms this intuition. Expected default costs and weighted average spreads are both lower for higher trigger ratios.\footnote{For example, if the conversion ratio is held at $\pi = 1.07$, and the trigger ratio increases from 5% to 7%, the expected size of equity issuance increases from about 0.7% to 8%.

The results also suggest that at the higher trigger, the commitment to preemptively issue equity and avoid conversion will be stronger even for less dilutive conversion ratios.

3.1.5 Comparative statics for other parameters

Table 2 presents comparative statics for parameters of the model that describe the characteristics of bank assets. The parameter values are varied around base case reported in Table 1. The table shows that with higher asset volatility ($\sigma$) and higher asset payout ratio ($\alpha$), the bank will issue less equity while holding the CoCo terms constant.\footnote{Note that a higher asset payout ratio implies lower risk-neutral drift in the asset value (calculated as $r - \alpha$) and higher dividend yield.}
these results is that higher volatility (or higher asset payout ratio) increases the equity option value of debt overhang in the post-conversion, producing weaker incentives to preempt triggering. As a policy suggestion: if regulators mandate banks to hold CoCo bonds, conversion terms should be more dilutive for banks that hold riskier assets and the banks that have higher dividend payout yield.

Comparative statics also demonstrate that incentives to issue equity are reduced for longer debt maturity, leading to higher default costs and wider spreads. This is because a longer horizon increases the expected future period over which the shareholders will find themselves in a “debt-overhang” regime (post-conversion of CoCos).

The effect of jumps on the bank’s incentives is more complicated. Jumps introduce skewness in the asset return distribution and a “fat” tail for negative returns. The higher the probability of jumps (higher value for parameter $\lambda$), the higher will be the probability that the trigger is breached and the higher will be the probability that default occurs.\footnote{There is a countervailing factor in play here: with higher negative jumps, the drift is adjusted upward increasing the upside potential for the equity.}

As reported in Table 2, when jumps are more frequent, the bank will be less committed to issue equity and recapitalize.\footnote{For example, given the conversion rate of 1.1 and the probability of jumps of 2.25% per year, the expected equity issuance rate reaches the plateau at the level of 5.62%. In comparison, with lower jump probability of 1.75%, equity issuance rate plateaus at the level of 6.67%.} This is because with higher jump risk, the equity issuance strategy has a diminishing marginal impact on the bank’s ability to reduce risk of triggering. In contrast, with less frequent jumps, the bank can avoid triggering with higher degree of certainty when it raises capital. These results suggest that in the economic environment in which changes are driven more by sudden negative changes (i.e., financial crisis) and less by a continuous diffusion, the effectiveness of CoCo as a self-disciplining mechanism will weaken, and an increase in conversion ratio will have a diminished impact on the bank’s incentives to manage its capital structure. Note also that since the CoCo bond is junior, the CoCo
investors will receive zero value at default making CoCo bond riskier when jumps are more frequent.

Finally, Table 2 shows that with larger transaction costs of issuing equity (higher values for parameter $e_1$ and $e_2$), the bank will be less willing to issue equity leading to wider spreads and higher default costs. This prediction reiterates the point made in the previous paragraph that market frictions will reduce the ex-ante commitment of a bank to manage its capital ratios.

3.2 Trigger uncertainty and covenant considerations

In the following sections we will consider a number of contractual frictions and contract variations which allow us to evaluate additional issues that may affect the bank’s capital structure strategy and the pricing of CoCo bonds. In the baseline model, the conversion triggers at the first instant when the conversion ratio it tripped (either through a drift or a jump). In reality, there can be a failure in the trigger mechanism. We will discuss two types of trigger failure. The first is the “type I” error that the CoCo fails to trigger despite capital ratios having breached the contractually specified threshold. The second is the “type II” error that the CoCo prematurely triggers even though the bank’s capital is still above the threshold. We also will quantify the value of dividend payout restrictions imposed in the post-conversion period and their impact on incentives for the bank to pursue low leverage.

3.2.1 Delayed triggering weakens ex-ante incentives

There is a number of market frictions that allow the bank to temporarily operate below the trigger without conversion. First, the bank’s capital ratio can only be verified quarterly when it releases its quarterly accounting reports. Thus, in the period between reports, the bank’s capital ratio can drop below the trigger without activating the conversion. Second, if the regulatory oversight is poor, a distressed bank will likely have more incentives to
misrepresent its true financial condition and overstate its capital ratio and delay triggering the conversion.

In order to capture the effect of this friction, we modify the model by introducing the so-called “delayed trigger”. We assume that given that the bank’s capital ratio is below the trigger, the conversion event can happen with some probability described by a Poisson-arrival process with the arrival intensity $\mu$ per year. Specifically, if the bank’s ratio declines and remains below the trigger during any short time interval between $t$ and $t + dt$ ($dt \to 0$), the probability that the conversion occurs during this period is $\mu \cdot dt$, and the probability that the trigger does not occur is $1 - \mu \cdot dt$. In this setting, the bank’s capital ratio can breach the trigger multiple times and stay below the trigger for some duration of time without conversion. With the assumption of the Poisson-arrival process, the probability that the conversion occurs next period does not depend on the duration of time the bank’s capital ratio have already stayed below the trigger in the prior periods. $^{25}$ The average length of time the bank stays below the trigger ratio without converting is $\frac{1}{\mu}$, and the probability that the conversion happens within one year period is $1 - e^{-\mu}$. Note, in the base case model, the conversion arrival intensity is $\mu = \infty$, meaning no delays.

Figure 8 shows that if the probability (the arrival intensity) of the triggering declines, the bank’s incentive to issue equity preemptively weakens. For example, if the annualized probability of conversion is 75% and the bank has the CoCo tranche that specifies the dilutive conversion ratio of $\pi=1.07$, the bank is expected to issue equity at the rate of 0.5%. In comparison, for the base case (no delays), the bank equity issuance rate is expected to be three times as much. Default costs and weighted-average spreads are higher when delays are longer (i.e., when the conversion probability is lower). Weaker incentives are explained by the fact that with the “delayed trigger”, the likelihood of dilutive conversion will be lower.

$^{25}$ The arrival of the trigger event is independent from the Weiner process $W_V$ in the asset value returns.
And the longer the bank stays below the trigger ratio without activating the conversion, the lesser will be the need for preemptive recapitalizations.

These insights imply that a weaker enforcement of contractual terms diminishes positive effects of CoCo on bank incentives, thereby suggesting the value of enhanced bank disclosure and greater transparency.

3.2.2 Premature triggering weakens ex-ante incentives

There are several reasons for the “type II” trigger error, i.e., the situation when the conversion is activated even though the bank’s capital ratio is still above the pre-defined trigger. As we document in appendix 1, some recently issued CoCo contracts specify that banking regulators have a discretion to trigger the conversion. As such, there could be a possibility of premature intervention by overly risk-averse regulators. Another reason for premature triggering could be due to noisy market fluctuations or market manipulations.

In order to quantify the effect of premature triggering we modify the model by assuming that– in addition to the “hard trigger” at capital ratio of 6%, the CoCo conversion can be also triggered by regulators (or due to market manipulation) at capital ratios above 6%. It is plausible to assume that, everything being equal, regulators will more likely trigger prematurely if a bank has low capital (but still more than the pre-defined threshold). As such, we assume that the probability of premature triggering is negatively related to the bank’s capital ratio, \( cap.ratio = \frac{V-F-C}{V} \). In the model, the annual probability of the premature triggering is described by exponential probability function of bank capital ratio \( e^{(-cap.ratio) \cdot \varphi} \), where the bank capital ratio is still above the trigger, i.e., \( \frac{V-F-C}{V} > 6\% \), where \( \varphi \) is a positive constant.\(^{26}\)

Figure 9 depicts the analysis for the cases: two cases with more frequent and less

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\(^{26}\)Similarly to the setting in the previous section, the value of \( (cap.ratio) \cdot \varphi \) is a Poisson arrival intensity per year. The arrival of triggering event is independent from the random term \( W_V \).
frequent premature triggering described by parameter $\varphi=20$ and $\varphi=30$ respectively, as well as the base case which has no premature triggering. The upper-left chart in Figure 9 depicts the total probability of triggering (including premature triggering) as a function of bank capital. The model predicts that the bank that faces higher odds of premature trigger will be less incentivized to raise capital preemptively and reduce its leverage in the future. This is because the risk of a premature conversion weakens shareholders’ ability to control the risk of dilution.

This prediction resonates with the argument offered in the previous section that the more uncertainty (more noise) is in the CoCo terms the less valuable (from shareholders’ standpoint) will be the strategy of preemptive equity issuances. This result also re-emphasizes the design of trigger mechanisms that reduce such uncertainty.

### 3.2.3 Restricting dividend payouts post conversion enhances ex-ante incentives

Bank regulators discuss the idea of requiring banks to hold a tranche of contingent convertible debt in the capital structure at all times. If this mandate is implemented, banks will have to reissue a new CoCo bond every time the existing CoCo tranche is converted or paid off. To provide incentives for the bank to promptly reissue a new CoCo, bank regulators may require banks to retain earnings, restrict payouts and build capital during these periods. Alternatively, such restrictions can be introduced as debt covenants to protect senior bondholders by preventing distribution of capital to bank’s shareholders if the bank operates without CoCo in the capital structure.

In order to fully assess the impact of such requirements, we have to assume that the bank can choose the timing and the size of the new convertible bond that replaces the outgoing tranche. Technically, such setup will be difficult to implement within the framework of the model because of the need to expand the decision space (i.e., the need to incorporate the
bank’s decision on the size of the new CoCo tranche). Instead, we make the following simplifying assumption: After the CoCo conversion is triggered, the bank will face the restriction that it cannot pay dividends until maturity, and will have to retain all earnings to increase its capital base. The bank still can default on its existing senior debt if, after the conversion, bank assets decline to the default boundary. We assume that before the conversion, the bank is unconstrained to pay out its residual cash flow as dividends.

In Figure 10, we compare this case with the base case in which the bank is unconstrained. The payout restriction has an impact on the pricing of CoCo bond because such restriction will enhance ex-ante incentives to issue equity and raise more capital. With such restriction, the bank is expected to issue more equity for same level of dilution when compared to the base case, and as such, the default risk will decline incrementally due to enhanced incentives. For example, the model predicts that at the conversion ratio of 7% above par ($\pi = 1.07$), a payout restriction can incentivize the bank to raise equity in the future at the average rate of 6%. Without payout restrictions, expected future equity issuance rate is at 1.5%.

The explanation of this result is that after the conversion, the payout restriction reduces the debt overhang problem thereby creating more incentives to issue equity preemptively (relative to the unconstrained case). From the regulatory perspective, the model implies that the restriction imposed on capital distributions can strengthen incentive effects of CoCo bond, and, as such can be viewed as an additional tool that can help reduce risk of financial distress.

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27 Such extension of the model will increase the dimensionality of the problem and will make the stochastic control problem significantly more complex.
28 Technically, in the calculations, the drift and payout rate $\alpha$ of the asset process $V$ are adjusted to incorporate the restriction of no payouts.
3.3 Incentives to burn money

3.3.1 The effect of maturity on incentives to burn money

The bank will have incentive to burn money (burn assets or waste capital) and accelerate trigger only if at conversion, the bond’s principal will be (fully or partially) written down. The write-down of principal will generate windfall gains for the bank’s shareholders at the expense of CoCo investors. The incentive to “burn money” will depend on the remaining maturity of the bond and the level of bank capital. Theoretically, at the instant the bond comes due, the bank may be willing to burn as much capital as the amount of the write-down itself, conditional that the “burnt” amount will be enough to push it capital ratio below the trigger. For longer maturity, these incentives are more complex. When the remaining maturity is long, bank managers may find it optimal to wait, because burning money eliminates a valuable optionality that bank assets can increase in value and generate positive gains for the bank’s shareholders.

To quantify the impact of these incentives, we use the example of the Rabobank’s loss-absorbing bond that assumes 75% write-down of principal (See appendix 1 for details). In this model, an equivalent to Rabobank’s conversion terms is the conversion ratio ($\pi$) set at 0.25, which implies that $1 of par value of CoCo will convert to 25 cents of equity. In Figure 11, we vary the maturity of the bond and plot the amount the bank is expected to burn while keeping remaining parameter values at the base case level including the bank’s initial capital ratio (set at 8%). In order to disentangle the incremental impact of “money burning” incentives, we also calculate spreads and deadweight costs for the case where the bank is not allowed to burn money. The model implies that if (remaining) maturity is long and the bank’s capital ratio is relatively healthy, the option to burn money has a small impact on the initial bond prices. However, the option to burn money will be deeper “in-the-money” if the remaining maturity is short. Figure 11 shows that an instant before the
CoCo comes due, the bank (with an initial capital of 8%) is expected to burn as much as 2% of its assets to trigger the conversion. The intuition of this result is straightforward: An instant before maturity, the shareholders face two choices: 1) pay a full amount of principal, or 2) burn 2% of its assets and reduce its capital ratio from 8% to 6% which will breach the trigger and capture gains coming from 75% reduction of principal. The 75% write-down of principal generates gains to the shareholders valued at 3.75% of the banks assets.\textsuperscript{29} These gains exceed the amount of capital that the bank needs to burn. Figure 11 demonstrates that for a short maturity case, a two percentage point “spike” in expected deadweight costs reflects the expected amount that will be “burnt”.

### 3.3.2 Bank’s capital ratio and money burning

As we mentioned in the previous section, the extent to which the bank will “burn money” will depend on the remaining maturity of the bond and the bank capital ratio. Figure 12 plots the relation between bank’s capital ratio and the expected value of burnt money for three maturity cases: one day, one quarter, and a half year. In these tests, we vary the bank’s capital ratio and keep the size of CoCo and of the senior debt constant at the base case values. The expected amount of “money burning ” exhibits the inverse U-shaped relation with the bank capital ratio. Specifically, the expected size of burnt money will increase for the capital ratio for the range between 4% and 8% and declines for capital ratio of 8% and larger. The explanation of this non-monotonic relation is the following: When the banks capital ratio declines to a level very close to the conversion trigger, there is a high probability that the trigger will be breached simply because value of bank assets declines even more. In such cases, the bank has a lesser need to burn money to accelerate triggering, or if needed, it does not have to burn a large amount. Conversely, if the bank operates at a capital ratio

\textsuperscript{29} These gains are calculated as the product of $75\% \cdot 5\% = 3.75\%$, given that CoCo’s principal is 5% of assets.
that is significantly higher than the conversion trigger of 6%, the option to burn money will be farther out of the money, because it is unlikely that the bank’s capital ratio will drift close to the conversion trigger. In such cases, in order to breach the trigger, the bank will need to burn the amount that exceeds the value of expected gains. Therefore, at the capital ratio well above the conversion trigger, the bank is not expected to burn much money. In the middle range, at the capital ratio around 9%, the banks is expected to burn the largest amount of money. The expected deadweight costs exhibits a similar inverse U-shaped relation with the capital ratio.

One day before maturity, the bank will be willing to burn up to 3.75% of its capital (i.e., reduce its capital from up to 9.75% down to 6% trigger) in order to breach the trigger and generate gains from the 75% write-down of CoCo principal. At capital ratios above 9.75%, the bank does not burn assets because the amount it needs to burn will exceed the gains from the principal write-down.

3.3.3 Discussion of the write-down provision of the contract

As the model implies, the option to “burn money” is likely be in-the-money in times of crisis, when the bank has low capital. This write-down feature is thus highly undesirable, because it is exactly during the crisis that is important to have incentive structures in place that discourage banks to exercise value-destroying strategies. The possibility that the trigger will write-down the bond’s principal can also exacerbate other inefficiencies and value-reducing incentives associated with a typical debt overhang problem. For example, the incentive to accelerate the trigger will encourage the bank to increase risk even more, especially its downside risk. Also, the bank will be more incentivized to underinvest in positive NPV projects. These value-reducing incentives created by the write-down feature will be even worse than those generated by ordinary senior debt. Finally, due to the impact of money
burning incentives, the bank’s stock price and the price of CoCo bond will likely experience higher uncertainty near maturity date. This price uncertainty can create liquidity problems for the bank because near maturity, it will need to raise funds to pay off its maturing debt.

4 Conclusion

Contingent capital will likely play a role in the future regulatory landscape for banks and financial firms as it designed to help absorb losses during a financial crisis. This paper demonstrates that CoCos have consequences that go beyond mere loss absorbancy. We show that even a small tranche of CoCos can generate sizable incentives for banks, and these incentives can either create or destroy value, depending on their design.

Our main finding is the following: If the conversion terms of the CoCo are dilutive for the pre-existing shareholders – meaning that one dollar of par value converts to more than one dollar of equity – then banks will commit to maintain a precautionary equity capital. These “incentive dividend” can materially increase valuation, reduce the costs of bank capital, and reduce the social welfare costs of financial distress. This pre-commitment to conservative capital structure would be priced by the market at time of issuance, benefitting both issuers and investors.

The model predicts that “trigger uncertainty” weakens the bank’s incentive to pursue low leverage. Trigger uncertainty can arise either because triggering is delayed (e.g., inattentive or poorly incentivized regulators), or because triggering is premature (e.g., because of premature regulatory intervention or strategic market manipulation by speculators). Either way, we show that trigger uncertainty weakens the bank’s incentives because it weakens the management’s ability to control dilution risk.

The model shows that – in contrast to the positive incentives created by dilutive conversion terms – conversion terms that write down principal for bondholders create perverse
incentives for the banks to pursue higher leverage and “money burning.” When the bank becomes overleveraged and its capital ratio is close to the trigger, the bank will find it optimal to burn capital to push its capital ratio below the trigger. By burning assets, the bank can breach the trigger, write-down CoCo’s principal and guarantee a windfall gains for shareholders. So far financial regulations (including Basel-III) do not appear to recognize that this loss-absorbing feature can have value-destroying incentive effects on bank behavior. Our results underscore the urgency of expanding the policy discussion to emphasize not just “loss absorbancy” but also incentives.

Substantial room for future research remains. We restricted our attention to common equity conversions, but the model can easily be extended to analyze conversion to preferred stock. There are at least two reasons such conversion design might be desirable. First, when market triggers are used and the stock price is near the trigger, the stock price is vulnerable to a “death spiral” caused by market manipulation. Converting to preferred stock could mitigate this risk. Second, fixed income investors may be reluctant to own CoCos because their mandate prohibits them from owning common equity. Converting to more debt-like securities in the capital structure might help to address this concern, too.
References


A Appendix 1: Existing bonds

In this section we briefly describe some of the “loss absorbing” bonds issued by European banks during 2009-2012. Lloyds Banking Group issued the first contingent capital security in late 2009 with roughly £8.5 billion of the so-called “Enhanced Capital Notes” (ECN). ECNs had terms eligible to qualify as lower Tier 2 capital upon their issue and will automatically convert into Ordinary Shares if the issuer’s published core tier 1 capital ratio falls below 5 per cent. For this bond, the conversion to equity will take place on par for par basis. There are bonds in existence for which triggering assumes the write-down of bond’s principal. For example, Rabobank issued 1.25 billion of Senior Contingent Notes in early 2010. These securities include a write down provision of 75% of the principal if the bank’s equity capital ratio falls below 7%, with the remaining 25% of the notes to be repaid in cash. Unicredit Group issued 500 million of a perpetual non-cumulative 9.375% Tier 1 in mid-2010. This security too has a writedown feature triggered if the total capital ratio reaches 6%.

In contrast to these bonds, regulators were given contractually-specified discretion to trigger the Intesa Sanpaolo 9.5% perpetual Tier 1 securities issued in fall 2010. These contain a loss-absorption feature under which the principal is written down if the bank’s total capital ratio falls below 6% or other minimum threshold specified by regulators. Similarly, in February 2011, Credit Suisse issued around CHF6 billion equivalent of high-trigger Tier 1 contingent capital notes. These securities will convert into bank’s equity if 1) the group’s reported Basel 3 common equity Tier 1 ratio falls below 7%, or 2) Swiss Financial Markets Authority FINMA determines that Credit Swiss “requires public sector support to prevent

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30 See Pitt, Hindlian, Lawson and Himmelberg (2011) for a detailed overview.
31 In January 2011, Rabobank issued another tranche of $2 billion of 8.375% perpetual non-cumulative capital securities which can be written down if the bank’s equity capital ratio falls below 8%.
it from becoming insolvent, bankrupt or unable to pay a material amount of its debts, or other similar circumstances.” Subsequently, in March 2012, Credit Suisse has opened the order book for a high-trigger contingent capital issue. The rate of the debt is above 7% and the deal is more than CHF250m. Under the terms, it will convert into equity if the bank’s Core Tier 1 ratio falls below 7% or if the bank is declared non-viable. Most recently, in April 2013, Barclays Bank sold $1B CoCo bond which can be written-off entirely if the bank’s core Tier 1 capital ratio drops below 7%.32 The Barclays CoCo will count as Tier 2 capital under Pillar 1, the minimum capital requirements set by global and European regulators.

The above bonds have two types of conversion terms: 1) conversion on a par-for-par basis; or 2) conversion that assumes writedown of bond principal. The trigger mechanisms are either 1) accounting-based, or 2) a hybrid accounting-regulatory-based. We will mimic these terms for the calibration of the CoCo contract in the baseline case, and will quantify the effects of different trigger mechanisms.

Contingent capital have not yet been issued by US banks. In part we suspect this is because capital concern are not as pressing in the US; US banks have raised large amounts of capital over the past few years due to the incentives provided by the desire to exit TARP. As a result, banking regulators in the US have had the luxury of more time to evaluate the desirability of contingent capital. Another reason could be the US tax code, under which loss absorbing securities may not be tax deductible.33

B Appendix 2: A simple one-period model

In this section, we use a simple one-period numerical example in order to illustrate incentives that the CoCo tranche can create for the bank with respect to issuing equity (burning money).

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32 At the end of 2012, Barclays' Core Tier 1 capital ratio was 10.9%.
33 For additional discussion, see Harvard Law School Forum on Corporate Governance and Financial Regulation (2011) and Castillo (2011).
In the example, we consider the bank that is financed with a tranche of CoCo bond and a senior debt. The CoCo bond and the senior debt have principal values of $C = 20$ and $F = 60$ respectively, and both mature next period at $t = T$. As shown on Figure 13, at the current time $t = T - 1$, the bank’s assets are valued at $V_{T-1} = 87$, but next period they are expected to drift to either $88$ or $86$ with equal probabilities. For simplicity, we assume that the risk-free rate is zero and investors are risk-neutral. The conversion trigger is breached when the bank’s capital ratio (measured as the ratio of $\frac{V-F-C}{V}$) declines below 8%. If the conversion trigger is breached at $t = T$, the CoCo bond will convert into the predefined value of the bank’s equity. Otherwise, the equityholders have to make principal payments to the CoCo holders and the holders of senior debt or default.

We consider two cases of CoCo design. In the first case, the conversion ratio will be dilutive to the existing shareholders of the bank, where the conversion ratio equals $1.1$. This conversion ratio means that conversion will take place at above par, i.e., $1$ of CoCo bond will convert to $1.1$ of equity. In the other case, we assume that the CoCo bond is loss-absorbing with the conversion ratio of $0.75$ i.e., corresponding to a 25% write-down of principal. At the current asset value of $87$, the bank’s capital ratio is above the conversion trigger, i.e., $\frac{V-F-C}{V} = \frac{87-20-60}{87} = 8.05\%$, which exceeds 8% trigger. If the asset values drifts down to the value of $86$ at time $T$, however, the CoCo will convert because the trigger will be breached, i.e., $\frac{86-20-60}{86} = 6.9\% < 8\%$.

Assume for now that bank’s shareholders cannot affect the value of its assets. For the dilutive CoCo, the value of the equity will be either $88 - 20 - 60 = 8$ at $t = T$, if the asset value increases to $88$; or $86 - 1.1 \cdot 20 - 60 = 4$, if asset value drifts down to $86$ and the CoCo converts. Thus, at time $T - 1$, the bank’s equity is valued at $6.0 = 0.5 \cdot (8 + 4)$. For the CoCo with the loss-absorbing conversion, the equity value at $t = T$ will be either $8$, if the asset value increases to $88$; or $11(=86 - 0.75 \cdot 20 - 60)$, if asset value declines
to $86. Thus, at time $T - 1$, the bank’s equity is valued at $9.5 = 0.5 \cdot (8 + 11)$.

Now consider how the CoCo tranche can affect the shareholders’ incentive to issue equity or burn money. For simplicity, assume that at time $T - 1$, the shareholders of the bank can choose to either issue a fixed amount of $2$ of equity, burn $2$ of assets, or do nothing. It is straightforward to see that if the conversion ratio is dilutive to the shareholders, they will choose to issue $2$ of equity at time $T - 1$ in order to avoid breaching the trigger at $t = T$. Particularly, with $2$ of new equity issued at time $T - 1$, the size of the bank’s assets increases to $89$, and the value of the bank’s assets will drift to either $90$ or $88$ at time $T$. At these values, there will be no breaching the trigger and the total value of the equity at time $T$ will be $10$ or $8$, respectively. As such, the equity value of the existing shareholders at time $T - 1$ is $7 = 0.5 \cdot (10 + 8) - 2$, where the last term accounts for $2$ of newly issued equity. This equity value is higher than for the case when the bank do not issue equity.

Conversely, for the CoCo bond that assumes 25% write-off, the bank’s shareholders will be better off by burning $2$ worth of bank’s assets at time $T - 1$ and forcing the conversion at time $T$ regardless of whether the asset value increases or decreases next period. As depicted in Figure 13, the total value of bank’s equity at time $T$ will be either $11 (= 86 - 0.75 \cdot 20 - 60)$ or $9 (= 84 - 0.75 \cdot 20 - 60)$, if asset value drifts from $85$ (after $2$ of assets are burnt) to $86$ and $84$, respectively. Thus at time $T - 1$, the equity value of the existing shareholders is $10 = 0.5 \cdot (11 + 9)$, which is a higher than the value of equity in the case where the bank cannot burn its assets.

C Appendix 3: Analytics of the model

C.1 Valuation of the bank’s equity

In this section, we present valuation of the bank’s equity for the case before $E(V, F + C, t)$, and after the conversion $E(V, F, t)$. The value of equity is a function of its asset value $V$; its
principal $F + C$, if no conversion took place in prior periods, or $F$, if the CoCo bond was converted before; and time $t (\leq T)$. The CoCo bond is described by parameters $\pi$, $\theta$ and the size of CoCo bond $C$. First, we describe the valuation of equity for the case in which the conversion already have taken place in prior periods. For this case, the bank continues paying interest of its senior debt $f$ and its par value $F$ at maturity. At maturity date $T$, the value of the the bank’s equity is

$$E(V, F, T) = \max(0, V - F).$$

(4)

Any time prior to maturity, using standard arbitrage arguments, the value of the equity $E(V, F, t)$ is given by the solution to the following partial integro-differential equation (PIDE):

$$\frac{\sigma^2 V^2}{2} E_{VV} + (r - \alpha - \lambda \cdot k) E_V + E_t + \alpha V - f - r E - \lambda \cdot \mathbb{E}^Q \{ E(Y, V, F, t) - E(V, F, t) \} = 0, E \geq 0,$$

where subscripts $t$ and $V$ denote partial derivatives and $\mathbb{E}^Q$ is the expectation operator under the risk neutral measure $Q$. In this equation, $\alpha V$ is the cash flow generated by the bank’s assets and $\alpha V - f$ is the dividend payout to the shareholders; and the last term represents the expected change in equity value due to jumps. Note that after the CoCo bond converts, the bank will not choose to issue equity because such transaction would dilute the shareholders’ value and will benefit holders of the senior debt at the shareholders’ expense. Also, without CoCo bond the bank will not choose to burn its assets.

Now, consider the valuation of the bank’s equity for the case if the CoCo was not con-
verted in the prior periods. At maturity $t = T$:

$$
E(V, F + C, T) = V - C - F, \quad \frac{V - F - C}{V} > \theta, \text{ no conversion,}
$$

$$
E(V, F + C, T) = \max\{0, (V - F - (\pi) \cdot C)\}, \quad \text{if } \frac{V - F - C}{V} < \theta \text{ and } V \geq F,
$$

$$
E(V, F + C, T) = 0, \text{ otherwise.}
$$

Before maturity, $t < T$, the bank makes its equity issuance as well as default choices. These choices maximize the market value of the bank’s equity, which is the present value of net cash flow to the equityholders. The solution involves determining the free boundary conditions that divide the state space $(V, \{F + C \text{ or } F\}, t)$ into the three regions that characterize the bank’s choices: the no equity issuance/no money burning region, the equity issuance region, the money burning region, the conversion region, and the default region.\(^{34}\)

In the no equity issuance/no money burning region, it is not optimal for the bank to issue equity capital (or burn money). In this region, the equity value $E(V, F + C, t)$ equals the instantaneous cash flow net of coupon payment plus the expected value of the equity at time $t + \Delta t$ calculated under the risk neutral measure $Q$:

$$
E(V, F + C, t) = [\alpha V - f - c]dt + e^{-r dt} \mathbb{E}^Q\{E(V_{t+dt}, F + C, t+dt)\}, \quad t + dt \leq T. \quad (6)
$$

In this region, the equityholders will not choose to issue equity (or burn money) and the following inequalities must hold for any $\Delta V$:

\(^{34}\)For brevity, we omit the discussion of the technical detail of boundary and “high contact” conditions that applied to the value function $E$. For details see Oksendal and Sulem (2007).
\[
\begin{align*}
\alpha V - f - c \, dt + e^{-rdt} E^Q (E(V_{t+dt} + \Delta V, F, C, t + dt) - \Delta V - TC < \\
[\alpha V - f - c \, dt + e^{-rdt} E^Q (E(V_{t+dt}, F, C, t + dt)), \\
\Delta V > 0, \text{ i.e., equity issuance is not profitable.}
\end{align*}
\]

\[
E(V - \Delta V, F, t) - (\pi) \cdot C < [\alpha V - f - c \, dt + e^{-rdt} E^Q (E(V_{t+dt}, F, C, t + dt)) \text{ s.t., } \frac{V - \Delta V - F - C}{V - \Delta V} < \theta, \Delta V > 0, \text{ i.e., "money burning" at the amount of } \Delta V \text{ that leads to an immediate conversion is not profitable.}
\]

where TC are transaction costs of raising equity, \( TC = e_1 \cdot V_t + e_2 \cdot \Delta V \). The value of the equity \( E(V, F + C, t) \) in the no equity issuance/no money burning region is given by the solution to the following PIDE:

\[
\frac{\sigma^2 V^2}{2} E_{VV} + (r - \alpha - \lambda \cdot k) E_V + E_t + \alpha V - f - c - r E - \lambda \cdot E^Q \{E(Y \cdot V, F, t) - E(V, F + C, t)\} = 0, \text{ for any } \frac{V - F - C}{V - \Delta V} > \theta, \ E > 0. \tag{8}
\]

In the equity issuance region, the value of the bank’s equity \( E(V, F + C, t), t < T, \) can be determined by maximizing the expected value of equity, over all equity issuance policies \( \Delta V \):

\[
E(V, F + C, t) = [\alpha V - f - c \, dt + e^{-rdt} E^Q (\max_{\Delta V > 0} [E(V_{t+dt} + \Delta V, F + C, t + dt) - \Delta V - TC])], \text{ for } t < T. \tag{9}
\]

The bank will raise new capital if the net benefit of raising capital exceeds the transaction costs \( TC \). This condition characterizes a region where equityholders raise equity and increase the size of the bank capital. In the money burning region, there has to be some non-zero amount of assets \( \Delta V^* > 0 \) to be burn that satisfies the following condition:

\[
E(V - \Delta V^*, F, t) - (\pi) \cdot C > E(V, F + C, t), \text{ s.t., } \frac{V - \Delta V^* - F - C}{V - \Delta V^*} = \theta, \Delta V^* > 0, \text{ for } t < T. \tag{10}
\]
In this region, the equity value is set at: \( E(V, F + C, t) = E(V - \Delta V^*, F, t) - (\pi) \cdot C \). In the CoCo conversion region, the bank’s capital ratio is below the trigger \( \theta \), and the following condition is held \( \frac{V - F - C}{V^*} \leq \theta \). In this region, the CoCo converts to equity at the dollar amount of shares \( (\pi) \cdot C \). In this region, the equity value for the existing stockholders is set:

\[
E(V, F + C, t) = E(V, F, t) - (\pi) \cdot C, \quad \text{if } V > F, \text{ and } E(V, F, t) - (\pi) \cdot C > 0. \tag{11}
\]

where \((\pi) \cdot C\) is the equity value that the CoCo holders take over after the conversion. The remaining fraction of equity value \( E(V, F, t) - (\pi) \cdot C \) belongs to pre-existing shareholders of the bank. The first inequality reflects the condition that the CoCo conversion can take place only if the bank’s assets are larger than the size of its debt. As we pointed out earlier, a large negative jump can instantly reduce the value of bank’s assets to some level \( V' \) well below the conversion trigger so that \( \frac{V' - F - C}{V'} < \theta \), but above the default level \( V' > F \). If the conversion ratio \( \pi \) is high, the conversion ratio is reduced from \( \pi \) to \( \pi'(>0) \) such that the value of the existing shareholders is wiped out, i.e., \( E(V', F + C, t) = 0 \), and the bank is taken over by the holders of contingent capital, where \( \pi' \) satisfies \( E(V', F, t) = (\pi') \cdot C \). In this region, the value of the CoCo holders will be \( E(V', F, t) \). In default region, the equity value \( E = 0 \), if

\[
E(V, F, t) = [\alpha V - f]dt + e^{-\gamma dt}\mathbb{E}^Q\{E(V_{t+dt}, F, t + dt)\} < 0, \quad t < T. \tag{12}
\]

### C.2 Valuation of the senior debt

To calculate the value of the senior debt, we need to consider the bank’s default strategy and the shareholders’ optimal strategy to raise capital. At maturity date \( T \), the value of the
bank's debt is

\[
\begin{align*}
D(V, F, T) &= F, \text{ if } V \geq F \text{ and } D(V, F, T) = (1 - DC) \cdot V, \text{ otherwise,} \\
\text{CoCo bond was converted in prior periods.} \\
D(V, F + C, T) &= F, \text{ if } V \geq F \text{ and } D(V, F + C, T) = (1 - DC) \cdot V, \text{ otherwise,} \\
\text{no prior CoCo conversion,}
\end{align*}
\]

(13)

where DC represents proportional default costs (0 ≤ DC ≤ 1). Any time prior to maturity \( t < T \), the value of debt satisfies:

\[
\begin{align*}
\sigma^2 V^2 \frac{D_{VV}}{2} + (r - \alpha - \lambda \cdot k)D_V + D_t + f - rD - \lambda \cdot E_t^Q \{ D(Y \cdot V, F + C, t) - D(V, F + C, t) \} &= 0, \\
\text{if } E(V, F + C, t) \geq 0, \text{ no prior CoCo conversion.} \\
\frac{\sigma^2 V^2}{2} D_{VV} + (r - \alpha - \lambda \cdot k)D_V + D_t + f - rD - \lambda \cdot E_t^Q \{ D(Y \cdot V, F, t) - D(V, F, t) \} &= 0, \\
\text{if } E(V, F, t) \geq 0, \text{ the CoCo converted in prior periods.}
\end{align*}
\]

(14)

In the conversion region, the value of the senior debt is

\[
D(V, F + C, t) = D(V, F, t), \text{ if } \frac{V - F - C}{V} \leq \theta.
\]

(15)

In the region where the bank issues equity and increases its capital by \( \Delta V \), the value of senior debt satisfies:

\[
D(V, F + C, t_+) = D(V + \Delta V, F + C, t).
\]

(16)

In the region where the bank "burns" assets by the amount of \( \Delta V^* \) and forces the conversion, the value of senior debt satisfies:

\[
D(V, F + C, t_+) = D(V - \Delta V^*, F, t), \text{ s.t., } \frac{V - \Delta V^* - F - C}{V - \Delta V^*} = \theta.
\]

(17)
In default region, the assets are transferred to the holders of the senior debt:

\[ D(V, F, t) = (1 - DC) \cdot V, \text{ if } E(V, F, t) = 0. \]  

(18)

### C.3 Valuation of the CoCo Bond

Because of lower priority, the CoCo holders recover zero value at default. To calculate the value of CoCo debt, we need to consider the bank’s default strategy and strategy to raise capital as well the possibility of conversion. At the debt maturity date \( T \), if conversion did not take place in prior periods, the value of the CoCo bond, \( Z(V, F + C, T) \) is

\[
\begin{align*}
Z(V, F + C, T) &= C, \quad \frac{V - F - C}{V} > \theta, \\
Z(V, F + C, T) &= \min\{(V - F), (\pi) \cdot C\}, \quad \text{if } \frac{V - F - C}{V} < \theta \text{ and } V \geq F, \\
Z(V, F + C, T) &= 0, \quad \text{otherwise.}
\end{align*}
\]

(19)

Any time prior to maturity \( t < T \), the value of CoCo bond satisfies:

\[
\frac{\sigma^2 V^2}{2} Z_{VV} + (r - \alpha - \lambda \cdot k) Z_V + Z_t + c - r Z - \lambda \cdot \mathbb{E}_t^Q \{ Z(Y_t \cdot V, F + C, t) - Z(V, F + C, t) \} = 0,
\]

\[
\text{if } E(V, F + C, t) \geq 0.
\]

(20)

At the conversion boundary, the value of the CoCo bond is

\[
Z(V, F + C, t) = \min\{E(V, F, t), (\pi) \cdot C\} \quad \text{if } \frac{V - F - C}{V} \leq \theta.
\]

(21)

For \( V' \), s.t., \( \frac{V' - F - C}{V'} \) < \( \theta \) and \( V' > F \), existing equity investors are wiped out, and the residual value is allocated to CoCo investors. The bank remains operational and is taken over by the holders of the CoCo bond. For such values of assets, the value of the CoCo holders will be \( Z(V', F + C, t) = E(V', F, t) \). In the region where the bank burns money by the amount of \( \Delta V' > 0 \) and forces the conversion, the value of CoCo debt satisfies:
\[ Z(V, F + C, t_+) = Z(V - \triangle V', F + C, t_+) = (\pi) \cdot C, \text{ s.t., } \frac{V - \triangle V' - F - C}{V - \triangle V'} = \theta. \quad (22) \]

In the region where the bank issues equity and increases its capital by \( \triangle V \) at \( t \), the value of CoCo bond satisfies:

\[ Z(V, F + C, t_+) = Z(V + \triangle V, F + C, t). \quad (23) \]

In the default region:

\[ Z(V, F + C, t_+) = 0. \quad (24) \]
Table 1: Base Case Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The volatility of bank assets</td>
<td>$\sigma = 5%$</td>
</tr>
<tr>
<td>Asset cash flow yield</td>
<td>$\alpha = 5%$</td>
</tr>
<tr>
<td>The risk-free rate</td>
<td>$r = 5%$</td>
</tr>
<tr>
<td>The proportional default costs</td>
<td>$DC = 50%$</td>
</tr>
<tr>
<td>The face value of CoCo bond</td>
<td>$C/V_0 = 5%$</td>
</tr>
<tr>
<td>The face value of straight debt</td>
<td>$F/V_0 = 87%$</td>
</tr>
<tr>
<td>The bank's initial leverage ratio</td>
<td>$(F + C)/V_0 = 92%$</td>
</tr>
<tr>
<td>The bank's initial capital ratio</td>
<td>$(V_0 - F - C)/V_0 = 8%$</td>
</tr>
<tr>
<td>Maturity of CoCo and senior debt</td>
<td>6 years</td>
</tr>
<tr>
<td>A conversion trigger</td>
<td>$\theta = 6%$ of capital</td>
</tr>
<tr>
<td>The conversion ratio $\pi$</td>
<td>Varied from 1.0 (conversion at par) to 1.1 (10% above par)</td>
</tr>
<tr>
<td>Transaction costs of raising equity</td>
<td>$e1 = e2 = 0.025%$</td>
</tr>
<tr>
<td>The jump size</td>
<td>$k = -0.5$, i.e., the jump size is uniformly distributed between 1 and 0, implying an average loss of 50%</td>
</tr>
<tr>
<td>The annual probability of jump</td>
<td>$\lambda = 2%$ per year</td>
</tr>
</tbody>
</table>
Table 2: Comparative statics
This table reports credit spreads and other values generated by the model for different parameter values varied around the base case reported in Table 1. The conversion ratio is the market value of equity to which $1 of CoCo bond will convert when the conversion trigger is breached. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future net equity issuances is measured ex-ante at time zero as % of the bank's initial assets.

<table>
<thead>
<tr>
<th>Parameter varied</th>
<th>The CoCo Conversion Ratio</th>
<th>Weighted-Average Spread of CoCo and senior Debt</th>
<th>Spread of Straight Debt (b.p.)</th>
<th>Spread of CoCo Bond (b.p.)</th>
<th>Expected default costs</th>
<th>Expected Size of Future Equity Issuance</th>
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</thead>
<tbody>
<tr>
<td>Base case</td>
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<tr>
<td></td>
<td>1.0</td>
<td>45</td>
<td>46</td>
<td>23</td>
<td>3.47%</td>
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<td></td>
<td>1.075</td>
<td>40</td>
<td>42</td>
<td>12</td>
<td>2.96%</td>
<td>2.01%</td>
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<td></td>
<td>1.1</td>
<td>33</td>
<td>32</td>
<td>43</td>
<td>1.83%</td>
<td>6.01%</td>
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<td>Volatility</td>
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<td>of bank</td>
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<tr>
<td>assets</td>
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<td>38</td>
<td>38</td>
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<td>0.00%</td>
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<td>1.075</td>
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<td>33</td>
<td>44</td>
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<td>4.06%</td>
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<td>44</td>
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<td>4.07%</td>
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<td>55</td>
<td>19</td>
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<td>Asset cash</td>
<td>4%</td>
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<td>37</td>
<td>37</td>
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<td>32</td>
<td>42</td>
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<td>1.77%</td>
<td>4.12%</td>
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<td>4 years</td>
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<td></td>
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<td>41</td>
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<td>Probability of</td>
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<td>43</td>
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<td>jumps</td>
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<td>41</td>
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<td>3.18%</td>
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<td>1.77%</td>
<td>5.64%</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td>33</td>
<td>32</td>
<td>43</td>
<td>1.77%</td>
<td>5.64%</td>
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<tr>
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<td>1.075</td>
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<tr>
<td></td>
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<td>23</td>
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</table>
Figure 1: Sample Path for Bank's Assets and CoCo Debt
Figure 2: Sample paths for bank’s assets for the processes with and without jumps.
Figure 3: Base case terms of CoCo bond and the case with a straight debt only

These figures show the model-generated credit spreads and other values as a function of the CoCo conversion ratio of the bank that has CoCo bond and senior debt in its capital structure. The conversion ratio is the market value of equity to which $1 of par value of the CoCo bond will convert when the trigger is breached. The size of the CoCo bond is 5% of the bank's initial assets and the size of senior debt is 87%. The bank's initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The CoCo conversion is triggered when the bank's capital ratio declines to 6% or below. The remaining parameter values are as in the base case reported in Table 1. For comparison, the table also presents the scenario when the bank is financed with straight debt of the same total size of 92% of total assets. The credit spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity expressed as % of the bank's unlevered assets. The expected value of future equity issuances is measured ex-ante at time zero as % of the bank's initial assets.
Figure 4: Comparison to the case where the bank cannot issue equity

These figures show the model-generated credit spreads and other values as a function of the CoCo conversion ratio: for two cases: 1) where the bank can issue equity (base case) and 2) the case where the bank cannot issue equity. The conversion ratio is the market value of equity to which $1 of par value of CoCo bond will convert when the conversion trigger is breached. The size of CoCo bond is 5% of the bank's initial assets. The CoCo conversion is triggered when the bank's capital ratio declines to 6% or below. The bank's initial total leverage ratio is 92%, i.e., its initial capital ratio is 8%. The remaining parameter values are as in the base case reported in Table 1. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future equity issuances is measured ex-ante at time zero as % of the bank's initial assets. Probabilities of default and triggering are calculated under the real world probability measure.
Figure 5: Size of the CoCo
These figures show the model-generated credit spreads and other values as a function of the CoCo conversion ratio for three different sizes of the CoCo tranche. The values are calculated for three cases where the size of senior debt and CoCo bond are both varied so that the initial total leverage of the bank is held at the base case level of 92%. The initial bank's capital ratio is at 8%. The conversion ratio is the market value of equity to which $1 of par value of CoCo bond will convert when the conversion trigger is breached. The CoCo conversion is triggered when the bank's capital ratio declines to 6% or below. The remaining parameter values are as in the base case reported in Table 1. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future equity issuances is measured ex-ante at time zero as % of the bank's initial assets.
Figure 6: Size of the CoCo tranche and the conversion price for which expected default costs are 2%.
The curve shows the CoCo conversion ratio and a size of the CoCo tranche for which expected default costs are predicted to be at 2% of bank initial assets. Above this indifference curve default costs are less than 2%, and below this curve default costs are greater than 2%. This curve is calculated given that the size of straight debt and of CoCo bond are both varied so that the initial leverage of the bank is held at the base case level of 92% and the initial bank's capital is held at 8%. The conversion ratio is the market value of equity to which $1 of par value of CoCo bond will convert when the conversion trigger is breached. The CoCo conversion is triggered when the bank's capital ratio declines to 6% or below. The remaining parameter values are as in the base case reported in Table 1. Expected default costs are measured as the difference of bank's unlevered assets minus total market value of debt and equity as % of the bank unlevered assets.
Figure 7: Conversion trigger ratio
These figures show the model-generated credit spreads and other values as a function of the CoCo conversion ratio for three different values of the trigger. The conversion ratio is the market value of equity to which $1 of par value of CoCo bond will convert when the conversion trigger is breached. The size of CoCo bond is 5% of the bank’s initial assets. The bank’s initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The remaining parameter values are as in the base case reported in Table 1. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future net equity issuances is measured ex-ante at time zero as % of the bank's initial assets.
Figure 8: "Delayed trigger"

These figures show the model-generated credit spreads and other values as a function of the CoCo conversion ratio for two cases with delayed trigger and the base case (no delay). With a trigger delay, the conversion does not necessarily happen immediately after the bank's capital ratio falls below the trigger of 6%, but instead the conversion occurs with some probability. For example, the probability of 75% means that the conversion is expected to take place with probability of 75% within a year time, given that the bank's capital ratio remains below the trigger. The conversion ratio is the market value of equity to which $1 of par value of CoCo bond will convert when the conversion occurs. The size of CoCo bond is 5% of the bank's initial assets. The bank's initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The remaining parameter values are as in the base case reported in Table 1. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future net equity issuances is measured ex-ante at time zero as % of the bank's initial assets.
Figure 9: Premature triggering in addition to "hard trigger"

These figures show the model-generated credit spreads and other values as a function of the CoCo conversion ratio for two cases which assume a possibility of premature triggering (e.g., by regulators) and for the base case (no premature triggering). In the two cases, regulators can activate the conversion prematurely even though the bank's capital ratio is still above the contractually defined trigger of 6%. The probability (the intensity) of premature triggering is assumed to be negatively related to bank capital. Charts present two levels of intensity of premature triggering. The upper-left chart depicts the total (annual) probability of triggering as a function of bank capital. Note, for all three cases, the conversion is immediately triggered at the bank capital ratio of 6% and below. The conversion ratio is the market value of equity to which $1 of par value of CoCo bond will convert when the conversion is activated. The size of CoCo bond is 5% of the bank's initial assets. The bank's initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The remaining parameter values are as in the base case reported in Table 1. The spread is the difference between the yield of the bond and the risk-free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of future net equity issuances is measured ex-ante at time zero as % of the bank's initial assets.
Figure 10: The effect of covenant that restricts the bank's dividend payout

These figures show the model-generated credit spreads and other values as a function of the CoCo conversion ratio for two cases: 1) the base case, and 2) the case where the bank, after the CoCo conversion is triggered, is required to retain earnings to rebuild capital and pay no dividends until maturity. The conversion ratio is the market value of equity to which $1 of par value of CoCo bond will convert when the conversion trigger is breached. The size of CoCo bond is 5% of the bank's initial assets. The CoCo conversion is triggered when the bank's capital ratio declines to 6% or below. The bank's initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The remaining parameter values are as in the base case reported in Table 1. The spread is the difference between the yield of the bond and the risk-free rate. Expected default costs are measured as the difference of the bank’s unlevered assets minus total market value of debt and equity as % of the bank’s unlevered assets. The expected value of future net equity issuances is measured ex-ante at time zero as % of the bank's initial assets.
Figure 11: Expected size of money burning as a function of CoCo bond maturity
These figures depict the model-generated credit spreads and expected value of "money burning" that the bank will undertake in order to force the CoCo conversion as a function of the CoCo bond maturity. For comparison, the chart depicts values for the case in which the bank is not allowed to burn money. The CoCo contract assumes the conversion ratio of 0.25, implying the 75% write-down of the bond's principal at the conversion trigger. The size of CoCo bond is 5% of the bank's initial assets. The CoCo conversion is triggered when the bank's capital ratio declines to 6% or below. The bank's initial leverage ratio is 92%, i.e., its initial capital ratio is 8%. The remaining parameter values are as in the base case. The spread is the difference between the yield of the bond and the risk free rate. Expected default costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity as % of the bank's unlevered assets. The expected value of "money burning" is measured ex-ante as % of the bank assets.
Figure 12: Expected size of money burning as a function of the bank's capital ratio

These figures depict the model-generated credit spreads and expected value of "money burning" as a function of the bank's capital ratio (measured as $(V-C-F)/V$) for three maturity cases: one day (1/365 years), 0.25 year, and 0.5 year. In this graph, we vary the value of the bank capital and keep the size of CoCo and senior debt constant at the base case level. The CoCo contract assumes the conversion ratio of 0.25 which corresponds to 75% write-down of the principal at the triggering boundary. The CoCo conversion is triggered when the bank's capital ratio declines to 6% or below. The remaining parameter values are as in the base case reported in Table 1. The spread is the difference between the yield of the bond and the risk-free rate. Expected default and deadweight costs are measured as the difference of the bank's unlevered assets minus total market value of debt and equity reported as % of the bank's unlevered assets. The expected value of burnt money is measured ex-ante as % of the bank's unlevered assets.
**Figure 13: A one-period numerical example**

These charts depict assets value and the equity value of the bank at time T-1 and T. The bank is financed with the tranche of CoCo bond and a senior debt. The CoCo bond and the senior bond have principal values of $20 and $60 respectively, and both mature at period T. The conversion will be triggered if bank capital fall below 8%. If the conversion trigger is breached at t=T, CoCo bond will convert into the predetermined value of the bank's equity. Otherwise, the equity holders have to make principal payments the CoCo holders and the holders of senior debt. We consider two cases of CoCo design: 1) The CoCo assumes dilutive conversion price of $1.1, i.e., $1 of CoCo debt converts into $1.1 of equity; 2) the CoCo bond has a loss-absorbing feature with the conversion price of $0.75, i.e., 25% of the CoCo bond is written-down as soon as the conversion boundary is breached. At time t=T-1, the bank's assets are valued at $87, but next period they expected to drift to either the level of $88 or $86 with equal probabilities. Discount rate is zero and investors are risk-neutral. We consider three cases for decision that the bank can take at t-1: 1) bank's shareholders cannot affect the value of its assets; 2) the bank can choose to either 1) issue $2 of equity, or 2) "burn money" in the amount of $2 and reduce its capital ratio, or 3) do nothing.

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<tr>
<th>Assets</th>
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<th>$88</th>
<th>Cap. Ratio= 9.09% (No Conversion)</th>
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<td>$87</td>
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<td></td>
</tr>
<tr>
<td>prob=0.5</td>
<td></td>
<td>$86</td>
<td>Cap ratio=6.97% (CoCo converts)</td>
</tr>
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<td>$87</td>
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**Dilutive Conversion price: $1 of CoCo converts to $1.1 of equity**

<table>
<thead>
<tr>
<th>Assets process after bank issues $2 of equity</th>
<th>Equity value if the bank cannot issue new equity</th>
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</thead>
<tbody>
<tr>
<td>t=T-1</td>
<td>t=T</td>
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<tr>
<td>Assets</td>
<td>Equity value for existing shareholders</td>
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<tr>
<td>S$89</td>
<td>S$8=88-20-60 (No Conversion)</td>
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<tr>
<td>+2</td>
<td>S$4=86-(1.1)20-60 (CoCo converts)</td>
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<tr>
<td>87</td>
<td>S$7</td>
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<tr>
<td></td>
<td>S$6=88-20-60 (No Conversion)</td>
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**25% of CoCo principal is written off, i.e., $1 of CoCo converts to $0.75 of equity**

<table>
<thead>
<tr>
<th>Assets after bank &quot;burns money&quot; and reduces the size of its assets by $2</th>
<th>Equity value if the bank cannot &quot;burn money&quot;</th>
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<tr>
<td>t=T-1</td>
<td>t=T</td>
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<tr>
<td>Assets</td>
<td>Equity value for existing shareholders after they burn $2 of assets</td>
</tr>
<tr>
<td>S$87</td>
<td>S$8=88-20-60 (No Conversion)</td>
</tr>
<tr>
<td>-2</td>
<td>S$11=86-(0.75)20-60 (CoCo converts)</td>
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<td>S$85</td>
<td>S$10</td>
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<td>S$9=84-(0.75)20-60</td>
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